TECHNICAL REPORT 106

Total and Merchantable Volume Equations for Common Tree Species in British Columbia

By Region and Biogeoclimactic Zone

2016

BRITISH COLUMBIA

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Gordon D. Nigh

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ISBN 978–0–7726–7010–6 – Print version
ISBN 978–0–7726–7011–3 – Digital version

Citation
Nigh, G.D. 2016. Total and merchantable volume equations for common tree species in British Columbia: by region and biogeoclimactic zone. Prov. B.C., Victoria, B.C. Tech. Rep. 106. www.for.gov.bc.ca/hfd/pubs/Docs/Tr/Tr106.htm

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Crown Publications, Queen's Printer
2nd Floor, 563 Superior Street
Victoria, BC V8W 9V7
1–800–663–6105
www.crownpub.bc.ca

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ABSTRACT

The British Columbia Ministry of Forests, Lands and Natural Resource Operations maintains tree taper equations that can be used to estimate individual tree volume; however, the calculations involved in making these estimates are non-trivial. A simple tree stem volume equation overcomes this issue and is easily programmed, for example, in a spreadsheet. Volume equations were developed in 1976. These equations show some bias when tested against the available stem analysis data. Therefore, four new sets of volume equations were developed: total volume by species and region (coast or interior); total volume by species and biogeoclimatic zone; merchantable volume by species and region (coast or interior); and merchantable volume by species and biogeoclimatic zone. Merchantable tree volume is total tree volume minus the volume of the stump and top. These equations provide more accurate volume estimates than the 1976 volume equations and the taper equations.

ACKNOWLEDGEMENTS

I thank Rene de Jong with the British Columbia Ministry of Forests, Lands and Natural Resource Operations for providing the volume estimates from Kozak's taper equations for the comparison between volumes from the new volume equations and Kozak's taper equations. I thank Peter Ott, David Stuart, and Wenli Xu with the Ministry of Forests, Lands and Natural Resource Operations for review comments.

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INTRODUCTION

The British Columbia Ministry of Forests, Lands and Natural Resource Operations maintains a set of taper equations that allow inside bark diameter to be estimated at any point along the stem of a tree (Kozak 2004, 2009). Taper equations can be used to calculate both total tree stem volume and merchantable stem volume from diameter at breast height outside bark (dbh) and total tree height by conceptually sectioning the stem into small sections, using Smalian's formula (Husch et al. 1972) to calculate the volume of each section, and summing the volume associated with each section. With this method, the taper equation is used to estimate the diameter at the bottom and top of each section. The stump and top-most section are assumed to be cylindrical and conical, respectively. Stem volume can also be calculated using the volume of revolution integration technique (Greenspan and Benney 1973), but this technique is not explored here because it is not used within the Ministry. While both of these methods are accurate, they are computationally intensive and cannot easily be coded in a spreadsheet, which may be expedient if, for example, the spreadsheet contains dbh and total tree height for many trees and the stem volume of these trees is required. Calculating the volume with other software packages such as SAS (SAS Institute Inc. 2011) or R (R Core Team 2013) is easier than in a spreadsheet but is still non-trivial, especially for the volume of revolution technique, which may require numerical integration.

Volume equations similar to the ones being proposed here already exist (B.C. Ministry of Forests 1976). These equations predict inside bark volume. Very little information is given about the data used to fit these models, but it appears that the data were from stem analysis. In total, 34 641 trees were used in the fitting. A larger data set that is suitable for developing volume equations is now available.

The objective of this project was to first test the 1976 volume equations for bias because they may be satisfactory for continued usage. The second objective was to develop volume equations for major commercial species (Table 1) that can easily be programmed into a spreadsheet. Equations were developed by species and region (coast or interior) and by species and Biogeoclimatic Ecosystem Classification (BEC) zone (Meidinger and Pojar 1991). The volume equations predict both total volume and merchantable volume from dbh and height. Merchantable volume is the total volume of a tree minus the stump and top volume, where stump height is 0.3 m and the top is the part of the stem that is above the point where the stem has an inside bark diameter of 4 cm. The merchantable volume equations are independent of minimum diameter thresholds because trees that are less than the minimum can just have their merchantable volume set to zero.

TABLE 1 *Major commercial species for which volume equations were developed. Equations were developed by species and region, and by species and Biogeoclimatic Ecosystem Classification (BEC) zone. Sample sizes are given in parentheses. Note that equations were developed only for zones with 30 or more observations. Therefore, the sum of the sample sizes by species and zone does not equal the sum of the samples sizes by species and regions in cases where there is a zone with fewer than 30 observations.*

Species	Scientific name	Species code	Region[a]	BEC zones[b]
Black cottonwood	*Populus trichocarpa*	Ac	C (121), I (922)	BWBS (605), CWH (117), ICH (95), IDF (170), SBS (41)
Trembling aspen	*Populus tremuloides*	At	I (2875)	BWBS (1831), ICH (44), IDF (116), SBS (868)
Balsam	*Abies amabilis, A. grandis*	B	C (2565)	CWH (2385), MH (175)
Subalpine fir	*Abies lasiocarpa*	Bl	I (7936)	AT (39), BWBS (312), ESSF (2712), ICH (1242), IDF (86), MS (230), SBS (3057), SWB (234)
Western redcedar	*Thuja plicata*	Cw	C (2023), I (4478)	CWH (1998), ESSF (250), ICH (3931), IDF (148), MS (147)
Red alder	*Alnus rubra*	Dr	C (689)	CWH (678)
Paper birch	*Betula papyrifera*	Ep	I (524)	BWBS (163), ICH (85), IDF (93), SBS (162)
Douglas-fir	*Pseudotsuga menziesii*	Fd	C (1251), I (4357)	CDF (32), CWH (1213), ESSF (284), ICH (829), IDF (1886), MS (420), PP (393), SBS (538)
Western hemlock	*Tsuga heterophylla*	Hw	C (6438), I (7134)	CWH (6277), ESSF (562), ICH (6419), IDF (38), MH (161), SBS (113)
Larch	*Larix occidentalis, L. laricina*	L	I (1330)	BWBS (205), ESSF (100), ICH (348), IDF (263), MS (412)
Bigleaf maple	*Acer macroyphyllum*	Mb	C (141)	CWH (141)
Whitebark pine	*Pinus albicaulis*	Pa	I (36)	ESSF (36)
Lodgepole pine	*Pinus contorta*	Pl	C (66), I (7331)	BWBS (1426), CWH (66), ESSF (735), ICH (476), IDF (916), MS (666), SBPS (861), SBS (2131), SWB (120)
Western white pine	*Pinus monticola*	Pw	C (38), I (369)	CWH (37), ICH (313), IDF (53)
Ponderosa pine	*Pinus ponderosa*	Py	I (647)	ICH (135), IDF (318), PP (192)
Sitka spruce	*Picea sitchensis*	Ss	C (994)	CWH (992)
Spruce	*Picea glauca, P. engelmannii, P. glauca* x *engelmannii, P. mariana*	S	C (10002)	BWBS (2824), ESSF (1583), ICH (1415), IDF (301), MS (681), SBPS (142), SBS (2675), SWB (381)
Yellow cedar	*Chamaecyparis nootkatensis*	Yc	C (675)	CWH (657)

a C: coastal, I: interior.

b AT: Alpine Tundra, BWBS: Boreal White and Black Spruce, CDF: Coastal Douglas-fir, CWH: Coastal Western Hemlock, ESSF: Engelmann Spruce – Subalpine Fir, ICH: Interior Cedar – Hemlock, IDF: Interior Douglas-fir, MH: Mountain Hemlock, MS: Montane Spruce, PP: Ponderosa Pine, SBPS: Sub-Boreal Pine – Spruce, SBS: Sub-Boreal Spruce, SWB: Spruce – Willow – Birch.

DATA

The data for this project came from the taper and Net Volume Adjustment Factor (NVAF) projects. Data collection for taper equations has effectively been replaced by the NVAF program. Information about NVAF data collection is provided in B.C. Ministry of Forests, Lands and Natural Resource Operations (2011). Trees from both of these projects were selected, felled, and sectioned. Sectioning points were taken at stump height (0.3 m), 0.6 m, 0.9 m, breast height (1.3 m or 1.37 m for some of the trees sampled before breast height was metricated), and 10 equally spaced intervals along the remainder of the stem. However, many exceptions to these sectioning standards are present in the data. Inside bark diameter (dib) was measured at each sectioning point. Diameter at breast height, total height, and tree age were also recorded. Trees with forks were not used in this analysis.

Biogeoclimatic Ecosystem Classification information was collected for each tree. The BEC zone was of particular importance because volume equations were developed by zone and because BEC zone was used to classify the trees as being coastal (the CDF, CWH, and MH zones [see Table 1 for full names of the BEC zones]) or interior (the remaining zones). Table 1 shows the sample sizes by species and region, and by species and BEC zone. Volume equations were not developed for species and regions or species and BEC zones that did not have at least 30 observations because the sample size was too small to obtain reliable equations.

The volume of the stump was calculated by assuming that the stump is a cylinder with a height of 0.3 m and a diameter of $dib_{0.3}$, where $dib_{0.3}$ is inside bark diameter at stump height. The volume of each section except the bottom and top sections was calculated with Smalian's formula, which is the average of the inside bark area at the bottom and the top of the section multiplied by the length of the section. The stem cross-section is assumed to be circular for the purpose of calculating the inside bark area. The volume of the top section is calculated by assuming that the top is a cone with a base equal to the dib of its bottom and a height equal to its length. Total tree volume is obtained by summing the volume of each section, including the stump and top sections. Merchantable volume is calculated by subtracting the volume of the stump and the volume of the stem above the point where the tree has a 4-cm dib. This point usually does not lie at a sectioning point. Therefore, it must be found by linearly interpolating dib at the bottom and top of the section within which dib is equal to 4 cm and conceptually adding this point as another sectioning point. The volume of the top above 4-cm dib can now be calculated as was done with other sections. If a tree had a stump dib less than 4 cm, then merchantable volume was not calculated. There were nine trees where the stump height measurement was not at 0.3 m. These trees were not used in the analysis for merchantable volume since stump volume could not be calculated.

ANALYSIS

Three analyses were done to meet the objectives of this project: testing the 1976 volume equations (B.C. Ministry of Forests 1976), developing new volume equations, and comparing the volume estimates from the new equations to those from the taper equations. All statistical tests were done at a significance level of $\alpha = 0.05$.

Testing the 1976 Volume Equations

All of the 1976 volume equations are of the form:

$$v = b_0 \times \mathrm{dbh}^{b_1} \times ht^{b_2} \tag{1}$$

where v = whole stem volume (m^3), dbh = diameter at breast height (cm), ht = total tree height (m), and b_i, $i = 0, 1, 2$ are model parameters. These models were fit with linear regression after a logarithmic (base 10) linearization of the equation. Reduction factors that convert total stem volume into merchantable volume are included in the 1976 publication. Volume equations were produced for each species and, where appropriate, for Forest Inventory Zones (FIZs) A, B, and C (roughly corresponding to BEC zones CDF, CWH, and MH) and FIZs D to L (corresponding to the interior of British Columbia). For some species, separate equations were developed for FIZ K and L, which roughly correspond to BEC zones BWBS and SWB. Furthermore, for some species, volume equations were developed for immature (≤ 120 years) and mature (> 120 years) trees. Volume was predicted with the 1976 volume equations for each tree in the data set. Selection of the appropriate equation to estimate tree volume was based on species, tree age, and BEC zone. Each volume equation was tested by calculating the error in the predicted volume (actual volume minus predicted volume), then calculating the mean error and using a t test to test whether the mean error is significantly different from zero. This test was performed for all the observations for each equation to check for overall bias. The data were also binned into 0.5-m^3 classes based on observed volume, with bin midpoints at 0.25, 0.75, 1.25 m^3, and so on, to check for bias across the volume range of the equations. A step-down Bonferroni adjustment proposed by Holm (Bretz et al. 2011) was done for the binned analyses to control the experiment-wise error rate of $\alpha = 0.05$. Only bins with 20 or more observations were included in this analysis to obtain reasonably accurate estimates of bias.

Developing New Volume Equations

The new volume equations have the same functional form as the 1976 volume equations; that is, Equation 1. This function is often chosen by other researchers (e.g., Fowler 1997; Pillsbury et al. 1998; Gonzalez-Benecke et al. 2014) and is a generalization of the volume equation with parameters b_1 and b_2 fixed at 2 and 1, respectively.

A preliminary analysis with nonlinear least-squares regression (procedure NLIN in SAS [SAS Institute Inc. 2011]) showed that parameter b_0 had significant parameter-effects nonlinearity (Ratkowsky 1983). By re-parameterizing b_0 so that it enters into the volume equation as an exponent with base e, the parameter-effects nonlinearity is reduced to acceptable levels. This re-parameterization does not affect the effective parameter estimate, but it does affect

its standard error, and hence affects any test or confidence interval that is constructed for the parameter.

Equation (2) was fit to the data using maximum likelihood (procedure NLMIXED in SAS [SAS Institute Inc. 2011]):

$$v_i = e^{b_0} \times \mathrm{dbh}_i^{b_1} \times ht_i^{b_2} + \varepsilon_i \tag{2}$$

where v_i is whole stem volume or merchantable volume (m^3) for tree i, dbh_i is the diameter at breast height (cm) of tree i, ht_i is the height (m) of tree i, b_j, $j = 0, 1, 2$ are parameters of the equation, and ε_i is a random error term for tree i. The standard assumptions are assumed for the error term except that a preliminary analysis showed that the variance of the equations increased with predicted volume. Therefore, the variance of the error term was modelled as a power of the mean covariance structure in the fitting process (Pinheiro and Bates 2000):

$$\mathrm{var}(\varepsilon_i) = \sigma^2 \times pv_i^{\delta} \tag{3}$$

where σ^2 is a scaling factor, pv_i is the predicted volume for tree i, and δ is a parameter to be estimated. Procedure NLMIXED instead of NLIN was used to fit the equations so that this variance function could be modelled.

The studentized residuals were calculated as the residual (actual volume minus predicted volume) divided by its standard error; that is, the square root of $\mathrm{Var}(\varepsilon_i)$ given by Equation 3. The assumptions about the random error term ε_i were tested with the studentized residuals. The zero mean assumption was tested with a t test, and the normality assumption was tested with the W-test (Shapiro and Wilk 1965) and with q–q plots (Mason et al. 1989). The W-test is valid only when the number of observations does not exceed 2000. Also, the W-test is very sensitive to departures from normality for large sample sizes (Mason et al. 1989). Therefore, the q–q plots were used in conjunction with the formal test to confirm normality of the residuals. The residuals were plotted against predicted volume to check for constant variance. These plots also revealed 22 trees to be outliers, which were subsequently removed from the analysis. These outlying trees were compared to trees in the data set that had similar diameters and were deemed to have had unusual heights and/or had volumes that were unusual when compared to trees with similar diameters and heights.

Testing the New Volume Equations

The fitted volume equations were tested for bias with the same procedure that was used to test the 1976 volume equations. The new volume equations were also tested against the current Ministry of Forests, Lands and Natural Resource Operations method of estimating volume with the Kozak (2004, 2009) taper equations. This was done by calculating the mean error in the volume predictions from both the volume equations developed here and from the taper equations by species and BEC zone and comparing these mean errors. Eight trees with heights greater than 75 m were not included in this comparison due to limitations in the compiler that calculates volume from the taper equations.

RESULTS

All results are shown to three decimal places. Therefore, values that are closer to zero than ± 0.0005 are displayed as ± 0.000.

Testing the 1976 Volume Equations

Table 2 presents the results of the testing of the 1976 volume equations. The average error is shown by volume class (bin) and the overall average error. Average errors that are significantly different from zero are in bold font. Note that many of the average errors that are significantly different from zero are quite small; the large sample sizes cause the average errors to be significant. Many of the 1976 volume equations are statistically significantly biased when tested against the current data set. These equations also show some bias when examined by tree volume bin, with the equations for interior western redcedar and western hemlock and interior subalpine fir (except for the BWBS and SWB zones) showing bias over a large range of tree volume bins.

Fitting and Testing Volume Equations by Species and Region

Table 3 contains summary statistics for the trees in the fitting data set by species and region. The results of the fitting of the total volume equations by species and region are provided in Tables 4 and 5. Table 4 contains the estimates of the model parameters and their standard errors and estimates of δ and σ^2 (for the power of the mean error term given by Equation 3). The mean of the residuals was close to zero for all equations, and none were significantly different from zero. Of 217 volume classes (bins) that were available for testing for bias across the range of trees sizes (recall that only classes with 20 or more observations were tested), 25 classes showed bias that was significantly different from zero. These classes and the mean of the residuals for each class are shown in Table 5. Most of the bias occurs in the small tree volume classes, and the bias is small.

The results of the fitting of the merchantable volume equations by species and region are provided in Tables 6 and 7. Table 6 contains the estimates of the model parameters and their standard errors and estimates of δ and σ^2. The mean of the residuals was close to zero for all equations, and none were significantly different from zero. Of 217 volume classes (bins) that were available for testing for bias across the range of tree sizes, 20 classes showed bias that was significantly different from zero. These classes and the mean of the residuals for each class are shown in Table 7. Most of the bias occurs in the small tree volume classes, although unlike the equations for total volume by species and regions, some large volume classes have significant bias.

Fitting and Testing Volume Equations by Species and Biogeoclimatic Ecosystem Classification Zone

Table 8 contains summary statistics for the trees in the fitting data set by species and zone. The results of the fitting of the total volume equations by species and zone are provided in Tables 9 and 10. Table 9 contains the estimates of the model parameters and their standard errors and estimates of δ and σ^2. The mean of the residuals was close to zero for all equations, and none were significantly different from zero. Of 361 volume classes (bins) that were available for testing for bias across the range of trees sizes, 30 classes showed bias that was significantly different from zero. These classes and the mean of the residuals for each class are shown in Table 10. Most of the bias occurs in the small tree volume classes, and the bias is small.

TABLE 2 *Results of the testing of the B.C. Ministry of Forests (1976) volume equations. This table contains the mean error by volume class and the overall error for all volume classes. Errors that are significantly different from zero are in a bold italic font. Means are not presented for cells with fewer than 20 observations. The regions are as follows: A: all of the province, A–BS: all of the province except the BWBS and SWB zones, C: coast, C≤120: coast and age≤120, C>120: coast and age>120, I: interior, I–BS: interior except the BWBS and SWB zones, BS: only the BWBS and SWB zones. See Table 1 for definitions of the species and Biogeoclimatic Ecosystem Classification zone codes.*

		Volume class (m^3)																									
Species	**Region**	**0.25**	**0.75**	**1.25**	**1.75**	**2.25**	**2.75**	**3.25**	**3.75**	**4.25**	**4.75**	**5.25**	**5.75**	**6.25**	**6.75**	**7.25**	**7.75**	**8.25**	**8.75**	**9.25**	**9.75**	**10.25**	**10.75**	**11.25**	**11.75**	**12.25**	**All**
Ac	A–BS	−0.003	0.001	−0.001		−0.026		−0.012																			***−0.058***
	BS	−0.004	0.000	−0.006	0.015	0.002																					0.004
At	A–BS	***−0.004***	−0.004	0.007	−0.039																						***−0.003***
	BS	***−0.006***	−0.002	***0.023***	0.031	***0.054***	0.049																				***0.006***
B	C	***−0.008***	***−0.024***	***−0.019***	***−0.036***	−0.022	−0.008	−0.009	−0.022	−0.044	−0.106	0.028	0.033	−0.164	−0.335	−0.118	−0.339	0.042	−0.680	−0.034		0.088	−0.165				***−0.036***
Bl	I–BS	***−0.005***	***−0.013***	***−0.015***	***−0.041***	***−0.043***	***−0.076***	−0.041	0.052	0.078	−0.024																***−0.010***
	BS	−0.000	0.008	−0.021	−0.025																						0.000
Cw	C≤120	0.001	***0.019***	−0.004	−0.044																						***−0.034***
	C>120	−0.005	0.006	0.005	0.009	0.008	−0.006	0.000	0.047	−0.026	0.018	−0.184	0.120	0.061	0.014	0.050	−0.099										***0.173***
	I	***−0.002***	***−0.015***	***−0.037***	***−0.067***	***−0.089***	***−0.131***	***−0.159***	***−0.149***	***−0.251***	***−0.265***	***−0.442***	***−0.386***	***−0.421***	***−0.601***	***−0.605***	***−0.510***	***−1.107***	***−0.805***	***−1.036***	***−1.048***	−0.362	−0.524		***−1.090***		***−0.192***
Dr	A	−0.002	−0.000	***−0.040***	***−0.064***																						***−0.012***
Ep	A–BS	***−0.005***	−0.005																								***−0.007***
	BS	***−0.005***																									***−0.005***
Fd	C≤120	***−0.005***	−0.014	−0.002	−0.017																						***−0.030***
	C>120	−0.010	−0.034	−0.042	−0.024	−0.033	−0.123	0.001	0.015	0.163	−0.036	−0.048				0.017											***0.136***
	I	***−0.005***	−0.006	−0.010	0.009	0.026	0.015	0.047	0.091	0.099	***0.190***	0.147	0.240	0.128													***0.020***
Hw	C≤120	***−0.003***	0.002	−0.006	***−0.074***	***−0.114***	−0.186																				***−0.044***
	C>120	***−0.005***	***−0.014***	−0.011	0.002	0.021	0.020	***0.069***	0.052	0.014	0.040	0.082	***0.198***	0.171	0.137	−0.079	0.101	0.273	0.338	0.192	0.398	−0.094	0.386	−0.166	0.097	−0.042	***0.038***
	I	***−0.016***	***−0.035***	***−0.043***	***−0.052***	***−0.083***	***−0.118***	***−0.128***	***−0.154***	***−0.298***	***−0.217***	***−0.378***	***−0.356***	***−0.474***	***−0.351***	***−0.720***	***−0.439***										***−0.075***
L	I–BS	−0.002	0.006	−0.014	−0.039	−0.009	−0.111	−0.008	−0.080																		−0.013
	BS	0.000																									−0.000
Mb	C	***−0.004***																									***−0.004***
Pa	A																										***0.125***
Pl	A–BS	***0.004***	***0.014***	−0.001	−0.017	−0.013																					***0.005***
	BS	−0.000	0.000	−0.020	−0.040																						***−0.002***
Pw	A	−0.002	−0.001	0.000	−0.043	−0.029																					−0.018
Py	I	***−0.012***	−0.014	−0.022	0.010	0.049	0.026																				***0.060***
S	I–BS	***−0.007***	***−0.006***	0.005	−0.002	−0.016	−0.024	−0.045	−0.057	−0.043	−0.009	−0.193	−0.177	−0.284	−0.471												***−0.022***
	BS	***−0.002***	0.004	−0.012	−0.006	0.021	0.001	−0.030																			−0.001
Ss	C≤120	−0.005	0.012	***0.039***	−0.003		0.020																				−0.012
	C>120		−0.020	−0.026	0.010	0.051		−0.123	0.117		***0.370***	0.284															***0.439***
Yc	A	0.000	0.005	0.044	0.067	0.041	0.035	0.029	0.189																		***0.186***

TABLE 3 *Summary statistics for the trees in the fitting data set by species and region (C: coast, I: interior). See Table 1 for definitions of the species codes.*

		dbh (cm)			Height (m)			Age (yr)		
Species	**Region**	**Average**	**Minimum**	**Maximum**	**Average**	**Minimum**	**Maximum**	**Average**	**Minimum**	**Maximum**
Ac	C	53.6	14.1	113.0	39.1	19.9	52.8	70	23	169
	I	35.0	6.6	136.9	26.7	9.1	48.9	86	18	321
At	I	25.9	3.8	72.9	22.4	4.4	38.3	89	13	254
B	C	45.2	10.4	148.2	30.6	4.0	63.4	243	32	746
Bl	I	27.8	3.9	96.0	20.8	3.2	53.3	156	20	479
Cw	C	52.6	7.7	246.0	26.8	5.9	63.5	214	20	944
	I	49.5	3.3	209.6	26.3	3.3	54.9	202	28	882
Dr	C	24.3	6.4	59.2	21.6	8.2	40.8	36	9	129
Ep	I	20.5	5.1	53.4	19.7	8.0	27.2	80	30	296
Fd	C	57.3	5.4	216.4	34.3	6.3	76.7	202	14	647
	I	37.3	2.1	147.6	23.6	2.6	54.3	158	24	560
Hw	C	43.7	5.2	190.7	28.7	5.0	67.1	227	12	990
	I	37.9	3.8	135.3	25.7	3.2	50.4	197	17	695
L	I	34.6	4.1	103.3	26.8	4.2	53.7	160	16	704
Mb	C	17.4	7.1	41.9	18.2	11.4	30.0	40	18	83
Pa	I	41.7	15.2	60.4	22.6	11.7	28.9	260	119	402
Pl	C	23.2	4.1	41.7	18.2	5.3	33.2	97	20	227
	I	24.2	3.8	70.0	20.4	3.1	40.8	126	15	365
Pw	C	42.9	8.2	116.6	28.8	11.6	58.1	197	52	375
	I	40.0	7.1	105.4	30.8	6.8	53.7	134	42	310
Py	I	39.3	3.1	112.7	21.0	3.2	48.7	178	24	542
S	I	31.5	4.1	122.3	23.9	3.0	55.8	152	16	839
Ss	C	68.2	3.1	371.7	38.3	3.3	82.3	177	10	852
Yc	C	44.8	8.4	203.0	22.5	4.3	49.0	311	67	993

TABLE 4 *Results of the total volume equation fitting by species and region (C: coast, I: interior). Shown are the estimates of the total volume equation parameters and their standard errors (below in parentheses) and the estimates of the error parameters δ and σ^2. See Table 1 for definitions of the species codes.*

Species	Region	b_0	b_1	b_2	δ	σ^2
Ac	C	−10.079	1.766	1.148	2.250	0.005
		(0.146)	(0.039)	(0.075)		
	I	−10.507	1.768	1.250	2.038	0.007
		(0.030)	(0.010)	(0.016)		
At	I	−10.314	1.863	1.135	2.044	0.007
		(0.015)	(0.006)	(0.010)		
B	C	−9.857	1.786	1.108	1.946	0.008
		(0.014)	(0.009)	(0.010)		
Bl	I	−9.879	1.843	1.025	1.961	0.008
		(0.008)	(0.005)	(0.006)		
Cw	C	−9.497	1.724	1.027	2.082	0.012
		(0.017)	(0.009)	(0.012)		
	I	−9.506	1.728	1.012	2.049	0.010
		(0.011)	(0.006)	(0.009)		
Dr	C	−10.170	1.866	1.103	2.033	0.007
		(0.031)	(0.012)	(0.018)		
Ep	I	−10.210	1.911	1.038	2.217	0.011
		(0.061)	(0.017)	(0.030)		
Fd	C	−9.988	1.709	1.159	2.064	0.012
		(0.026)	(0.011)	(0.016)		
	I	−10.096	1.755	1.146	2.029	0.011
		(0.012)	(0.006)	(0.008)		
Hw	C	−10.036	1.828	1.088	1.974	0.012
		(0.011)	(0.005)	(0.006)		
	I	−10.174	1.888	1.056	1.895	0.009
		(0.011)	(0.005)	(0.007)		
L	I	−9.912	1.761	1.069	2.079	0.014
		(0.027)	(0.015)	(0.019)		
Mb	C	−10.351	1.867	1.122	1.891	0.005
		(0.121)	(0.029)	(0.058)		
Pa	I	−9.940	1.878	1.037	2.181	0.004
		(0.150)	(0.075)	(0.112)		
Pl	C	−10.022	1.865	1.064	2.313	0.009
		(0.055)	(0.044)	(0.057)		
	I	−9.920	1.887	1.018	1.871	0.007
		(0.011)	(0.005)	(0.005)		
Pw	C	−10.005	1.876	1.047	1.962	0.007
		(0.123)	(0.070)	(0.099)		
	I	−9.960	1.780	1.120	2.219	0.005
		(0.036)	(0.022)	(0.030)		
Py	I	−10.465	1.980	1.035	1.798	0.016
		(0.038)	(0.021)	(0.024)		
S	I	−9.951	1.807	1.080	1.971	0.008
		(0.007)	(0.004)	(0.005)		
Ss	C	−9.842	1.820	1.036	2.134	0.010
		(0.027)	(0.011)	(0.017)		
Yc	C	−9.689	1.931	0.878	2.231	0.012
		(0.024)	(0.014)	(0.018)		

TABLE 5 *Volume classes from the total volume equations by species and region analysis that have residuals with a mean that is significantly different from zero. Definitions of the species codes are provided in Table 1; C: coast, I: interior.*

Species	Region	Volume class (m^3)	Residual mean (m^3)
Ac	I	0.25	–0.005
At	I	0.25	–0.005
		1.25	0.021
		2.25	0.063
B	C	0.25	–0.005
		0.75	–0.014
Bl	I	0.25	–0.003
Cw	C	2.75	–0.104
Fd	I	0.25	–0.006
		0.75	–0.010
		1.25	–0.017
Hw	C	0.25	–0.003
		0.75	–0.009
	I	0.25	–0.007
		0.75	–0.007
L	I	1.75	–0.067
Pl	I	0.75	0.004
		1.25	–0.019
		1.75	–0.050
		2.25	–0.076
Py	I	0.25	–0.009
S	I	0.25	–0.005
		1.25	0.009
Ss	C	0.25	–0.008
Yc	C	2.75	–0.127

TABLE 6 *Results of the merchantable volume equation fitting by species and region (C: coast, I: interior). Shown are the estimates of the merchantable volume equation parameters and their standard errors (below in parentheses) and the estimates of the error parameters δ and σ^2. See Table 1 for definitions of the species codes.*

Species	Region	b_0	b_1	b_2	δ	σ^2
Ac	C	−10.238	1.759	1.193	2.233	0.005
		(0.149)	(0.040)	(0.077)		
	I	−10.740	1.766	1.312	2.025	0.007
		(0.032)	(0.010)	(0.017)		
At	I	−10.829	1.860	1.287	1.570	0.006
		(0.023)	(0.007)	(0.011)		
B	C	−9.982	1.763	1.160	1.938	0.008
		(0.015)	(0.009)	(0.011)		
Bl	I	−10.132	1.830	1.107	1.836	0.008
		(0.009)	(0.006)	(0.006)		
Cw	C	−9.693	1.716	1.079	2.056	0.013
		(0.018)	(0.009)	(0.013)		
	I	−9.750	1.696	1.105	1.970	0.012
		(0.013)	(0.007)	(0.010)		
Dr	C	−10.435	1.867	1.173	1.960	0.007
		(0.034)	(0.013)	(0.019)		
Ep	I	−10.679	1.940	1.147	2.023	0.009
		(0.071)	(0.020)	(0.033)		
Fd	C	−10.168	1.704	1.208	1.996	0.013
		(0.027)	(0.011)	(0.017)		
	I	−10.388	1.770	1.207	1.860	0.012
		(0.014)	(0.006)	(0.008)		
Hw	C	−10.200	1.822	1.134	1.936	0.013
		(0.012)	(0.005)	(0.007)		
	I	−10.387	1.885	1.114	1.896	0.010
		(0.011)	(0.006)	(0.007)		
L	I	−10.059	1.748	1.113	1.755	0.016
		(0.040)	(0.016)	(0.023)		
Mb	C	−10.731	1.898	1.198	1.738	0.004
		(0.121)	(0.031)	(0.061)		
Pa	I	−10.170	1.860	1.116	2.095	0.005
		(0.166)	(0.079)	(0.119)		
Pl	C	−10.284	1.873	1.126	1.575	0.006
		(0.117)	(0.053)	(0.060)		
	I	−10.041	1.880	1.051	1.586	0.006
		(0.013)	(0.005)	(0.006)		
Pw	C	−10.168	1.874	1.086	1.901	0.008
		(0.136)	(0.076)	(0.107)		
	I	−10.189	1.755	1.203	2.118	0.006
		(0.045)	(0.024)	(0.035)		
Py	I	−10.809	2.021	1.080	1.749	0.017
		(0.037)	(0.022)	(0.024)		
S	I	−10.223	1.781	1.178	1.903	0.008
		(0.007)	(0.005)	(0.005)		
Ss	C	−10.051	1.794	1.112	2.040	0.013
		(0.036)	(0.012)	(0.020)		
Yc	C	−9.957	1.936	0.941	2.163	0.014
		(0.028)	(0.015)	(0.020)		

TABLE 7 *Volume classes from the merchantable volume equations by species and region analysis that have residuals with a mean that is significantly different from zero. Definitions of the species are provided in Table 1; C: coast, I: interior.*

Species	Region	Volume class (m^3)	Residual mean (m^3)
Ac	I	0.25	–0.003
B	C	0.25	–0.005
		0.75	–0.012
Bl	I	0.25	–0.001
		1.75	–0.025
Cw	C	2.75	–0.105
		3.25	–0.138
		5.25	–0.368
Fd	I	0.25	–0.002
		1.25	–0.018
Hw	I	0.25	–0.005
		4.25	–0.134
Pl	I	0.75	0.007
		1.25	–0.014
		1.75	–0.045
		2.25	–0.072
S	I	0.25	–0.002
		6.75	–0.486
Ss	C	0.25	–0.007
Yc	C	2.75	–0.133

TABLE 8 *Summary statistics for the trees in the fitting data set by species and zone. See Table 1 for species and zone codes.*

		dbh (cm)			Height (m)			Age (yr)		
Species	**Zone**	**Average**	**Minimum**	**Maximum**	**Average**	**Minimum**	**Maximum**	**Average**	**Minimum**	**Maximum**
Ac	BWBS	36.2	8.9	100.3	27.9	12.8	46.1	97	18	220
	CWH	51.8	14.1	90.4	39.0	19.9	52.8	69	23	169
	ICH	24.0	6.6	136.9	19.3	9.1	41.8	41	18	154
	IDF	30.7	13.0	128.9	25.4	13.5	48.9	54	20	208
	SBS	61.1	24.5	122.2	33.3	19.9	40.9	158	79	321
At	BWBS	28.5	9.4	72.9	24.2	10.3	38.3	98	27	254
	ICH	17.2	7.9	39.6	19.8	10.9	31.0	54	36	96
	IDF	12.0	4.1	52.0	11.9	4.4	29.8	43	14	211
	SBS	22.9	3.8	65.9	20.3	5.2	33.9	77	13	199
B	CWH	45.0	10.4	148.2	30.5	4.0	63.4	244	39	746
	MH	47.7	12.5	111.8	30.9	4.0	53.3	238	32	535
Bl	AT	33.0	20.7	48.6	25.3	20.4	32.7	155	93	202
	BWBS	29.9	13.4	56.9	21.9	5.5	29.8	145	72	368
	ESSF	28.0	3.9	96.0	20.0	3.3	53.3	164	20	441
	ICH	27.9	6.7	86.2	22.1	5.9	44.5	158	49	378
	IDF	20.4	5.4	53.2	16.0	4.4	37.6	97	47	248
	MS	24.5	13.0	62.4	19.6	8.5	37.4	122	56	286
	SBS	28.1	4.1	93.1	21.4	3.2	45.1	153	25	479
	SWB	25.3	13.0	64.1	18.1	7.0	32.0	146	82	216
Cw	CWH	52.5	7.7	246.0	26.8	5.9	63.5	214	20	944
	ESSF	43.2	13.0	127.3	25.1	7.4	46.0	184	75	493
	ICH	50.8	4.7	209.6	26.7	3.3	54.9	206	33	882
	IDF	36.3	3.3	172.4	21.3	4.1	49.6	129	28	453
	MS	39.4	13.0	123.6	21.3	7.3	42.8	219	54	360
Dr	CWH	24.3	6.4	59.2	21.6	8.2	40.8	36	9	129
Ep	BWBS	20.5	13.0	36.4	20.3	13.6	24.1	85	47	158
	ICH	21.5	11.3	40.2	20.5	14.7	25.6	70	44	176
	IDF	16.1	5.1	34.3	16.7	9.0	24.3	68	30	127
	SBS	22.4	13.0	53.4	20.6	13.7	27.2	88	50	296
Fd	CDF	56.0	14.9	130.5	34.1	17.4	53.0	129	37	410
	CWH	57.3	5.4	216.4	34.3	6.3	76.7	204	14	647
	ESSF	39.4	13.0	118.0	26.4	8.0	52.4	158	47	459
	ICH	40.7	10.0	123.2	28.4	5.7	51.7	144	44	560
	IDF	36.8	2.1	147.6	21.7	2.6	54.3	158	24	504
	MS	35.2	4.4	112.3	22.8	4.8	47.8	153	45	451
	PP	34.4	12.8	113.1	19.2	6.3	40.8	179	45	402
	SBS	35.6	4.6	119.6	25.4	4.9	44.7	162	38	462
Hw	CWH	43.6	5.2	190.7	28.6	5.0	67.1	225	12	990
	ESSF	40.4	13.0	105.7	27.6	6.3	49.1	217	55	480
	ICH	37.9	3.8	135.3	25.6	3.2	50.4	197	17	695
	IDF	26.1	14.0	52.4	19.4	8.3	33.7	133	45	212
	MH	48.2	13.5	115.2	30.2	8.8	49.1	295	66	944
	SBS	32.7	18.4	82.6	20.4	10.1	34.7	120	74	281
L	BWBS	20.2	8.2	43.5	17.6	8.4	28.2	123	59	208
	ESSF	46.1	15.3	91.3	33.8	19.4	46.4	221	75	445
	ICH	34.8	6.7	97.5	27.0	7.4	53.7	134	36	372
	IDF	30.4	4.1	89.7	24.6	4.2	50.9	129	16	486
	MS	41.6	12.3	103.3	30.9	13.5	50.2	204	63	704

TABLE 8 *Continued*

Species	Zone	dbh (cm)			Height (m)			Age (yr)		
		Average	Minimum	Maximum	Average	Minimum	Maximum	Average	Minimum	Maximum
Mb	CWH	17.4	7.1	41.9	18.2	11.4	30.0	40	18	83
Pa	ESSF	41.7	15.2	60.4	22.6	11.7	28.9	260	119	402
Pl	BWBS	23.3	4.3	50.7	19.9	6.1	30.8	121	19	245
	CWH	23.2	4.1	41.7	18.2	5.3	33.2	97	20	227
	ESSF	25.2	4.3	54.1	20.5	4.9	34.6	129	32	322
	ICH	24.7	12.5	54.3	22.9	10.5	39.7	115	41	256
	IDF	19.0	3.8	46.1	15.9	3.1	36.8	94	15	365
	MS	23.0	12.4	70.0	18.9	7.6	40.8	129	37	307
	SBPS	23.6	12.5	46.4	16.4	7.6	29.2	161	52	346
	SBS	27.1	4.1	60.0	24.2	4.3	37.5	126	25	298
	SWB	24.1	13.5	41.3	17.2	8.5	23.6	187	80	269
Pw	CWH	42.0	8.2	116.6	28.1	11.6	58.1	197	52	375
	ICH	38.7	7.1	105.4	30.3	6.8	53.7	124	42	310
	IDF	46.8	18.1	104.8	33.0	16.3	52.4	192	84	268
Py	ICH	55.8	14.2	112.7	32.2	6.2	48.7	210	40	379
	IDF	34.3	3.1	102.9	18.7	3.2	40.9	149	24	542
	PP	36.0	6.2	99.1	16.9	4.1	36.6	205	29	513
S	BWBS	27.2	8.1	77.6	22.0	7.0	43.7	147	32	389
	ESSF	31.5	4.1	110.7	22.1	3.0	51.8	153	16	839
	ICH	36.0	4.1	122.3	26.7	4.4	55.8	162	41	444
	IDF	25.4	5.3	88.4	20.6	4.6	51.2	102	30	273
	MS	34.8	7.4	87.7	26.7	6.3	52.1	166	50	376
	SBPS	33.6	13.3	64.3	24.8	9.1	38.8	228	59	441
	SBS	34.5	4.1	101.8	26.1	3.3	49.8	146	35	402
	SWB	23.8	13.0	48.0	17.4	10.0	27.7	173	60	354
Ss	CWH	68.2	3.1	371.7	38.3	3.3	82.3	177	10	852
Yc	CWH	44.7	8.4	203.0	22.4	4.3	49.0	307	67	993

TABLE 9 *Results of the total volume equation fitting by species and zone. Shown are the estimates of the total volume equation parameters and their standard errors (below in parentheses) and the estimates of the error parameters δ and σ^2. See Table 1 for definitions of the species codes and zones.*

Species	Zone	b_0	b_1	b_2	δ	σ^2
Ac	BWBS	−10.564	1.787	1.244	2.046	0.005
		(0.033)	(0.011)	(0.016)		
	CWH	−10.157	1.731	1.206	2.244	0.005
		(0.151)	(0.044)	(0.082)		
	ICH	−10.389	1.699	1.278	1.918	0.008
		(0.133)	(0.042)	(0.082)		
	IDF	−10.456	1.893	1.123	2.342	0.008
		(0.091)	(0.028)	(0.049)		
	SBS	−10.676	1.776	1.289	2.559	0.003
		(0.268)	(0.049)	(0.115)		
At	BWBS	−10.505	1.855	1.205	2.093	0.007
		(0.028)	(0.008)	(0.012)		
	ICH	−10.551	1.890	1.189	1.906	0.002
		(0.166)	(0.049)	(0.095)		
	IDF	−10.213	1.918	1.056	2.340	0.023
		(0.037)	(0.028)	(0.032)		
	SBS	−10.198	1.903	1.049	2.033	0.006
		(0.025)	(0.013)	(0.020)		
B	CWH	−9.855	1.786	1.106	1.959	0.008
		(0.015)	(0.009)	(0.011)		
	MH	−9.865	1.740	1.161	1.742	0.010
		(0.068)	(0.031)	(0.033)		
Bl	AT	−10.371	1.489	1.561	1.890	0.005
		(0.365)	(0.080)	(0.169)		
	BWBS	−9.896	1.726	1.163	1.853	0.004
		(0.060)	(0.022)	(0.029)		
	ESSF	−9.906	1.849	1.026	2.024	0.009
		(0.013)	(0.009)	(0.010)		
	ICH	−9.931	1.916	0.960	2.053	0.009
		(0.020)	(0.013)	(0.015)		
	IDF	−9.772	1.800	1.039	2.119	0.006
		(0.039)	(0.044)	(0.048)		
	MS	−9.997	1.833	1.066	1.993	0.006
		(0.050)	(0.028)	(0.033)		
	SBS	−9.790	1.813	1.033	1.834	0.007
		(0.013)	(0.009)	(0.009)		
	SWB	−9.903	1.741	1.144	1.990	0.005
		(0.050)	(0.023)	(0.033)		
Cw	CWH	−9.492	1.726	1.023	2.081	0.012
		(0.017)	(0.009)	(0.012)		
	ESSF	−9.462	1.685	1.054	2.004	0.007
		(0.047)	(0.026)	(0.034)		
	ICH	−9.489	1.718	1.017	2.048	0.010
		(0.012)	(0.007)	(0.009)		
	IDF	−9.668	1.836	0.933	2.037	0.010
		(0.053)	(0.028)	(0.042)		
	MS	−9.666	1.849	0.934	2.148	0.006
		(0.034)	(0.032)	(0.039)		

TABLE 9 *Continued*

Species	Zone	b_0	b_1	b_2	δ	σ^2
Dr	CWH	−10.167	1.867	1.101	2.046	0.007
		(0.031)	(0.012)	(0.018)		
Ep	BWBS	−10.976	1.876	1.327	2.597	0.011
		(0.161)	(0.031)	(0.066)		
	ICH	−10.389	1.921	1.102	2.529	0.015
		(0.250)	(0.042)	(0.090)		
	IDF	−9.919	1.964	0.882	2.131	0.007
		(0.109)	(0.036)	(0.066)		
	SBS	−10.561	1.935	1.123	2.277	0.011
		(0.145)	(0.031)	(0.057)		
Fd	CDF	−10.337	1.553	1.432	1.691	0.010
		(0.250)	(0.044)	(0.090)		
	CWH	−9.985	1.712	1.155	2.068	0.012
		(0.026)	(0.011)	(0.017)		
	ESSF	−10.103	1.757	1.139	1.956	0.009
		(0.052)	(0.025)	(0.033)		
	ICH	−10.119	1.712	1.203	2.006	0.009
		(0.030)	(0.015)	(0.019)		
	IDF	−10.073	1.774	1.115	2.045	0.012
		(0.018)	(0.009)	(0.013)		
	MS	−9.954	1.755	1.096	2.109	0.011
		(0.037)	(0.023)	(0.032)		
	PP	−10.086	1.813	1.072	2.200	0.014
		(0.042)	(0.019)	(0.025)		
	SBS	−10.190	1.730	1.206	1.946	0.011
		(0.039)	(0.020)	(0.027)		
Hw	CWH	−10.035	1.830	1.086	1.972	0.012
		(0.011)	(0.005)	(0.006)		
	ESSF	−10.331	1.941	1.039	1.898	0.009
		(0.037)	(0.020)	(0.024)		
	ICH	−10.162	1.885	1.056	1.899	0.010
		(0.011)	(0.006)	(0.007)		
	IDF	−10.513	1.759	1.300	1.620	0.004
		(0.120)	(0.082)	(0.083)		
	MH	−10.038	1.737	1.187	2.137	0.012
		(0.079)	(0.031)	(0.037)		
	SBS	−10.055	1.838	1.078	2.015	0.007
		(0.088)	(0.042)	(0.049)		
L	BWBS	−10.033	1.720	1.186	2.245	0.005
		(0.053)	(0.024)	(0.028)		
	ESSF	−9.490	1.729	0.978	2.055	0.012
		(0.208)	(0.050)	(0.088)		
	ICH	−10.209	1.800	1.120	2.230	0.010
		(0.042)	(0.021)	(0.028)		
	IDF	−10.069	1.781	1.089	2.133	0.016
		(0.034)	(0.037)	(0.044)		
	MS	−9.812	1.745	1.049	2.168	0.014
		(0.079)	(0.030)	(0.046)		
Mb	CWH	−10.351	1.867	1.122	1.891	0.005
		(0.121)	(0.029)	(0.058)		
Pa	ESSF	−9.940	1.878	1.037	2.181	0.004
		(0.150)	(0.075)	(0.112)		

TABLE 9 *Continued*

Species	Zone	b_0	b_1	b_2	δ	σ^2
Pl	BWBS	–9.853	1.898	0.995	2.015	0.004
		(0.021)	(0.009)	(0.012)		
	CWH	–10.022	1.865	1.064	2.313	0.009
		(0.055)	(0.044)	(0.057)		
	ESSF	–10.052	1.755	1.196	1.994	0.007
		(0.028)	(0.013)	(0.015)		
	ICH	–9.941	1.804	1.103	1.900	0.007
		(0.047)	(0.026)	(0.033)		
	IDF	–10.211	1.905	1.098	1.882	0.007
		(0.024)	(0.010)	(0.012)		
	MS	–10.032	1.811	1.137	1.885	0.008
		(0.042)	(0.017)	(0.019)		
	SBPS	–10.114	1.974	0.996	1.881	0.007
		(0.046)	(0.014)	(0.020)		
	SBS	–9.839	1.857	1.018	1.891	0.006
		(0.020)	(0.009)	(0.012)		
	SWB	–9.787	2.053	0.817	2.030	0.005
		(0.095)	(0.041)	(0.039)		
Pw	CWH	–10.017	1.874	1.053	1.977	0.007
		(0.130)	(0.072)	(0.105)		
	ICH	–9.934	1.771	1.120	2.240	0.005
		(0.036)	(0.023)	(0.031)		
	IDF	–10.386	1.706	1.331	1.761	0.007
		(0.163)	(0.070)	(0.112)		
Py	ICH	–10.529	1.825	1.232	1.860	0.013
		(0.114)	(0.042)	(0.056)		
	IDF	–10.519	1.979	1.046	1.793	0.015
		(0.058)	(0.036)	(0.045)		
	PP	–10.431	1.956	1.067	1.757	0.016
		(0.087)	(0.045)	(0.047)		
S	BWBS	–10.038	1.802	1.113	1.966	0.006
		(0.015)	(0.007)	(0.009)		
	ESSF	–9.834	1.810	1.042	2.011	0.008
		(0.013)	(0.010)	(0.012)		
	ICH	–9.879	1.784	1.084	2.089	0.008
		(0.017)	(0.013)	(0.016)		
	IDF	–10.038	1.841	1.068	1.810	0.008
		(0.042)	(0.040)	(0.046)		
	MS	–10.066	1.803	1.126	2.018	0.007
		(0.025)	(0.017)	(0.020)		
	SBPS	–10.108	1.841	1.086	1.808	0.004
		(0.075)	(0.042)	(0.054)		
	SBS	–10.024	1.801	1.107	1.910	0.009
		(0.017)	(0.010)	(0.013)		
	SWB	–9.970	1.804	1.075	2.131	0.005
		(0.050)	(0.020)	(0.025)		
Ss	CWH	–9.841	1.820	1.035	2.134	0.010
		(0.027)	(0.011)	(0.017)		
Yc	CWH	–9.686	1.930	0.878	2.236	0.012
		(0.024)	(0.014)	(0.018)		

TABLE 10 *Volume classes from the total volume equations by species and zone analysis that have residuals with a mean that is significantly different from zero. Definitions of the species and zone codes are provided in Table 1.*

Species	Zone	Volume class (m^3)	Residual mean (m^3)
At	BWBS	0.25	–0.005
	IDF	0.25	–0.004
	SBS	0.25	–0.003
B	CWH	0.25	–0.005
		0.75	–0.013
Bl	ESSF	0.25	–0.004
		0.75	–0.008
		3.75	0.232
		4.25	0.383
	ICH	0.25	–0.003
Cw	CWH	2.75	–0.105
Fd	IDF	0.25	–0.006
	MS	0.25	–0.006
	SBS	0.25	–0.011
Hw	CWH	0.25	–0.003
		0.75	–0.009
	ICH	0.25	–0.007
		0.75	–0.008
L	MS	0.25	–0.010
Pl	MS	0.25	–0.003
	SBPS	0.25	–0.003
		0.75	0.009
Py	IDF	0.25	–0.009
S	BWBS	0.25	–0.002
	ESSF	0.25	–0.004
	ICH	0.25	–0.005
	IDF	0.75	0.024
	MS	0.25	–0.004
	SBS	0.25	–0.007
Ss	CWH	0.25	–0.008

The results of the fitting of the merchantable volume equations by species and zone are provided in Tables 11 and 12. Table 11 contains the estimates of the model parameters and their standard errors and estimates of δ and σ^2. The mean of the residuals was close to zero for all equations, and none were significantly different from zero. Of 361 volume classes (bins) that were available for testing for bias across the range of trees sizes, 24 classes showed bias that was significantly different from zero. These classes and the mean of the residuals for each class are shown in Table 12. Most of the bias occurs in the small tree volume classes, although unlike the equations for total volume by species and zone, some large volume classes have significant bias.

Comparing the New Volume Equations to Volume Estimates Based on the Taper Equations

The results of the comparison between the total volume equations and the volumes estimated with the taper equations are provided in Table 13. The volume equations had a smaller mean error than the taper equations in more than 70% of the species and zones that were tested. Only about 13% of the volume equations had a mean error that was significantly different from zero, whereas more than 41% of the taper equation volumes had a mean error that was significantly different from zero. The volume equations are more accurate than the taper equations for estimating total volume.

DISCUSSION/CONCLUSION

This analysis resulted in the following volume equation for use with coefficients b_0, b_1, and b_2 given in Tables 4, 6, 9, and 11:

$$v = e^{b_0} \times \mathrm{dbh}^{b_1} \times ht^{b_2}$$

where v is tree volume (m^3), dbh is diameter at breast height (cm), and ht is total height (m). These volume equations predict total volume and merchantable volume by species and region and by species and biogeoclimatic zone. Merchantable volume is total volume minus the volume of the stem below stump height (0.3 m) and above the point where the stem has a diameter of 4 cm.

Testing of the 1976 volume equations shows that these equations have varying degrees of bias. The new volume equations are unbiased and are often more accurate than using the taper equations to estimate volume.

TABLE 11 *Results of the merchantable volume equation fitting by species and zone. Shown are the estimates of the merchantable volume equation parameters and their standard errors (below in parentheses) and the estimates of the error parameters δ and σ^2. See Table 1 for definitions of the species codes and zones.*

Species	Zone	b_0	b_1	b_2	δ	σ^2
Ac	BWBS	−10.740	1.784	1.290	2.036	0.005
		(0.034)	(0.011)	(0.016)		
	CWH	−10.316	1.723	1.251	2.229	0.005
		(0.155)	(0.045)	(0.084)		
	ICH	−10.814	1.663	1.437	1.757	0.007
		(0.139)	(0.043)	(0.084)		
	IDF	−10.666	1.891	1.178	2.334	0.009
		(0.096)	(0.030)	(0.052)		
	SBS	−10.840	1.768	1.336	2.582	0.003
		(0.269)	(0.050)	(0.116)		
At	BWBS	−10.681	1.850	1.254	2.093	0.007
		(0.028)	(0.008)	(0.012)		
	ICH	−10.994	1.894	1.314	2.054	0.003
		(0.165)	(0.052)	(0.097)		
	IDF	−10.874	1.927	1.237	1.455	0.006
		(0.137)	(0.046)	(0.079)		
	SBS	−10.724	1.896	1.208	1.456	0.005
		(0.047)	(0.016)	(0.025)		
B	CWH	−9.977	1.767	1.155	1.945	0.008
		(0.015)	(0.010)	(0.011)		
	MH	−10.024	1.722	1.217	1.793	0.010
		(0.068)	(0.031)	(0.032)		
Bl	AT	−10.560	1.467	1.632	1.910	0.005
		(0.369)	(0.081)	(0.171)		
	BWBS	−10.072	1.710	1.224	1.859	0.005
		(0.062)	(0.023)	(0.031)		
	ESSF	−10.256	1.850	1.124	1.838	0.009
		(0.014)	(0.010)	(0.011)		
	ICH	−10.132	1.916	1.010	2.015	0.010
		(0.022)	(0.014)	(0.016)		
	IDF	−10.197	1.767	1.203	2.042	0.006
		(0.046)	(0.048)	(0.053)		
	MS	−10.214	1.829	1.125	1.977	0.007
		(0.052)	(0.029)	(0.035)		
	SBS	−9.980	1.792	1.102	1.763	0.007
		(0.015)	(0.009)	(0.010)		
	SWB	−10.075	1.721	1.207	1.978	0.005
		(0.052)	(0.024)	(0.034)		
Cw	CWH	−9.688	1.718	1.074	2.055	0.013
		(0.018)	(0.009)	(0.013)		
	ESSF	−9.644	1.662	1.118	1.992	0.008
		(0.050)	(0.028)	(0.035)		
	ICH	−9.749	1.687	1.115	2.005	0.012
		(0.014)	(0.007)	(0.010)		
	IDF	−9.898	1.880	0.934	1.680	0.011
		(0.072)	(0.034)	(0.054)		
	MS	−9.870	1.830	1.000	2.160	0.006
		(0.034)	(0.032)	(0.039)		

TABLE 11 *Continued*

Species	Zone	b_0	b_1	b_2	δ	σ^2
Dr	CWH	−10.430	1.869	1.169	1.974	0.007
		(0.034)	(0.013)	(0.019)		
Ep	BWBS	−11.301	1.876	1.418	2.515	0.011
		(0.173)	(0.032)	(0.071)		
	ICH	−10.613	1.923	1.157	2.512	0.017
		(0.266)	(0.045)	(0.095)		
	IDF	−10.414	2.062	0.934	1.868	0.006
		(0.139)	(0.050)	(0.080)		
	SBS	−10.798	1.943	1.177	2.291	0.012
		(0.154)	(0.033)	(0.059)		
Fd	CDF	−10.459	1.548	1.465	1.712	0.010
		(0.253)	(0.045)	(0.091)		
	CWH	−10.167	1.707	1.204	1.997	0.013
		(0.028)	(0.011)	(0.017)		
	ESSF	−10.292	1.742	1.201	1.934	0.010
		(0.054)	(0.026)	(0.034)		
	ICH	−10.317	1.693	1.272	1.983	0.010
		(0.031)	(0.015)	(0.020)		
	IDF	−10.425	1.797	1.187	1.826	0.012
		(0.023)	(0.010)	(0.014)		
	MS	−10.248	1.788	1.137	1.809	0.011
		(0.047)	(0.025)	(0.034)		
	PP	−10.316	1.814	1.133	2.146	0.014
		(0.043)	(0.020)	(0.025)		
	SBS	−10.503	1.724	1.296	1.926	0.011
		(0.038)	(0.021)	(0.027)		
Hw	CWH	−10.200	1.824	1.131	1.933	0.013
		(0.012)	(0.005)	(0.007)		
	ESSF	−10.483	1.931	1.085	1.886	0.009
		(0.039)	(0.021)	(0.025)		
	ICH	−10.382	1.882	1.116	1.898	0.010
		(0.012)	(0.006)	(0.007)		
	IDF	−10.704	1.738	1.370	1.596	0.005
		(0.126)	(0.085)	(0.087)		
	MH	−10.191	1.725	1.236	2.141	0.013
		(0.083)	(0.032)	(0.040)		
	SBS	−10.245	1.828	1.139	1.983	0.007
		(0.094)	(0.044)	(0.052)		
L	BWBS	−10.315	1.717	1.266	2.208	0.006
		(0.055)	(0.026)	(0.030)		
	ESSF	−9.620	1.720	1.017	2.039	0.012
		(0.210)	(0.050)	(0.089)		
	ICH	−10.472	1.796	1.192	2.064	0.011
		(0.054)	(0.022)	(0.031)		
	IDF	−10.588	1.687	1.328	1.695	0.021
		(0.116)	(0.046)	(0.071)		
	MS	−9.959	1.736	1.092	2.161	0.015
		(0.080)	(0.030)	(0.047)		
Mb	CWH	−10.731	1.898	1.198	1.738	0.004
		(0.121)	(0.031)	(0.061)		
Pa	ESSF	−10.170	1.860	1.116	2.095	0.005
		(0.166)	(0.079)	(0.119)		

TABLE 11 *Continued*

Species	Zone	b_0	b_1	b_2	δ	σ^2
Pl	BWBS	–10.020	1.884	1.052	1.671	0.004
		(0.028)	(0.010)	(0.014)		
	CWH	–10.284	1.873	1.126	1.575	0.006
		(0.117)	(0.053)	(0.060)		
	ESSF	–10.280	1.749	1.262	1.764	0.006
		(0.038)	(0.015)	(0.017)		
	ICH	–10.098	1.798	1.146	1.867	0.008
		(0.050)	(0.027)	(0.035)		
	IDF	–10.642	1.938	1.196	1.467	0.005
		(0.048)	(0.013)	(0.015)		
	MS	–10.202	1.808	1.182	1.867	0.009
		(0.044)	(0.018)	(0.020)		
	SBPS	–10.305	1.979	1.043	1.849	0.007
		(0.048)	(0.015)	(0.021)		
	SBS	–10.013	1.843	1.076	1.693	0.006
		(0.025)	(0.010)	(0.013)		
	SWB	–9.922	2.062	0.840	2.023	0.005
		(0.098)	(0.042)	(0.040)		
Pw	CWH	–10.189	1.870	1.097	1.913	0.009
		(0.143)	(0.079)	(0.114)		
	ICH	–10.166	1.747	1.202	2.129	0.005
		(0.047)	(0.025)	(0.036)		
	IDF	–10.497	1.692	1.369	1.748	0.007
		(0.165)	(0.071)	(0.113)		
Py	ICH	–10.713	1.811	1.292	1.884	0.013
		(0.110)	(0.042)	(0.055)		
	IDF	–10.977	2.013	1.138	1.726	0.014
		(0.050)	(0.035)	(0.046)		
	PP	–10.698	1.980	1.111	1.803	0.017
		(0.081)	(0.044)	(0.048)		
S	BWBS	–10.233	1.791	1.172	1.948	0.006
		(0.016)	(0.008)	(0.010)		
	ESSF	–10.195	1.773	1.180	1.824	0.008
		(0.017)	(0.011)	(0.012)		
	ICH	–10.119	1.754	1.175	1.941	0.009
		(0.021)	(0.014)	(0.018)		
	IDF	–10.283	1.824	1.149	1.812	0.009
		(0.045)	(0.041)	(0.047)		
	MS	–10.271	1.796	1.183	2.003	0.008
		(0.026)	(0.018)	(0.020)		
	SBPS	–10.298	1.835	1.138	1.816	0.004
		(0.076)	(0.042)	(0.055)		
	SBS	–10.280	1.769	1.206	1.876	0.009
		(0.017)	(0.010)	(0.012)		
	SWB	–10.193	1.797	1.141	2.072	0.005
		(0.053)	(0.021)	(0.026)		
Ss	CWH	–10.050	1.795	1.111	2.040	0.013
		(0.036)	(0.012)	(0.020)		
Yc	CWH	–9.954	1.936	0.941	2.167	0.014
		(0.028)	(0.015)	(0.020)		

TABLE 12 *Volume classes from the merchantable volume equations by species and zone analysis that have residuals with a mean that is significantly different from zero. Definitions of the species and zone codes are provided in Table 1.*

Species	Zone	Volume class (m^3)	Residual mean (m^3)
At	BWBS	0.25	–0.005
B	CWH	0.25	–0.004
		0.75	–0.010
Bl	ESSF	0.75	–0.009
		4.25	0.252
Cw	CWH	2.75	–0.106
	ICH	8.25	–0.615
Fd	IDF	1.25	–0.028
	SBS	0.25	–0.007
Hw	ICH	0.25	–0.005
		4.25	–0.139
L	IDF	0.25	0.007
	MS	0.25	–0.010
Pl	ESSF	1.75	–0.076
	IDF	0.25	0.002
	MS	0.25	–0.003
	SBPS	0.25	–0.003
		0.75	0.008
S	BWBS	0.75	0.005
	ICH	0.25	–0.003
	IDF	0.75	0.026
	SBS	0.25	–0.005
Ss	CWH	0.25	–0.007
Yc	CWH	2.75	–0.129

TABLE 13 *Average errors in the volume estimates from the total volume equations and the Kozak (2004, 2009) taper equations. The average with the smallest magnitude is indicated by an asterisk (*), and the average errors that are significantly different from zero are in a bold italic font. Definitions of the species and Biogeoclimatic Ecosystem Classification (BEC) zone codes are provided in Table 1.*

		Volume estimate source	
Species	**BEC zone**	**Volume equation**	**Taper equation**
Ac	BWBS	–0.001*	***0.013***
	CWH	0.015*	0.043
	ICH	0.008*	***–0.030***
	IDF	***–0.098***	–0.030*
	SBS	–0.086	0.007*
At	BWBS	0.001*	***0.010***
	ICH	–0.001*	–0.002
	IDF	***–0.006***	–0.005*
	SBS	–0.001*	***0.004***
B	CWH	–0.008*	***0.042***
	MH	–0.008*	0.053
Bl	AT	–0.000*	0.012
	BWBS	–0.001*	***0.007***
	ESSF	***0.013****	***0.021***
	ICH	–0.004*	***0.015***
	IDF	0.004*	0.007
	MS	0.001*	***0.013***
	SBS	–0.002*	***0.005***
	SWB	–0.000*	***0.010***
Cw	CWH	0.018*	***0.187***
	ESSF	–0.003*	0.030
	ICH	***–0.042****	***0.080***
	IDF	0.030*	***0.074***
	MS	0.000*	0.045
Dr	CWH	–0.005	–0.000*
Ep	BWBS	0.000*	0.001
	ICH	–0.003	0.003*
	IDF	–0.001	–0.001*
	SBS	–0.004	–0.001*
Fd	CDF	–0.025*	–0.084
	CWH	0.015*	***0.119***
	ESSF	0.001*	0.031
	ICH	0.013*	***0.038***
	IDF	***0.013****	***0.018***
	MS	0.014*	***0.047***
	PP	0.009*	0.017
	SBS	0.003*	***0.023***
Hw	CWH	***–0.028****	***0.028***
	ESSF	–0.002*	***0.038***
	ICH	***–0.007****	***0.012***
	IDF	–0.000*	–0.008
	MH	–0.034	0.030*
	SBS	–0.004*	0.014

TABLE 13 *Continued*

Species	BEC zone	Volume estimate source	
		Volume equation	Taper equation
L	BWBS	0.000*	***0.003***
	ESSF	–0.022	0.012*
	ICH	–0.012	0.008*
	IDF	***–0.039***	–0.010*
	MS	–0.017	0.011*
Mb	CWH	0.001	0.000*
Pa	ESSF	–0.002*	0.012
Pl	BWBS	–0.002*	***0.002***
	CWH	–0.005	–0.000*
	ESSF	–0.003	0.001*
	ICH	–0.001*	0.005
	IDF	–0.000	0.000*
	MS	–0.000*	0.004
	SBPS	0.000*	0.002
	SBS	–0.002*	***0.004***
	SWB	–0.000*	0.001
Pw	CWH	–0.062	0.021*
	ICH	–0.013*	0.018
	IDF	0.002*	0.055
Py	ICH	0.026*	0.077
	IDF	0.031	0.029*
	PP	0.010	0.004*
S	BWBS	–0.002*	***0.009***
	ESSF	–0.000*	0.008
	ICH	***–0.020***	***0.020****
	IDF	–0.007	0.000*
	MS	–0.006*	***0.026***
	SBPS	0.000*	0.014
	SBS	–0.004*	***0.015***
	SWB	–0.000*	0.003
Ss	CWH	***–0.295***	***0.205****
Yc	CWH	–0.019*	***0.222***

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