Graphical Representations of Cartographic Uncertainty Across Different Spatial Scales

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ABSTRACT

With increasing emphasis on the production of modeled maps of various components of forest resources, the production of maps of model uncertainty needs greater attention. Typical measures of uncertainty include error statistics or scatterplots of model residuals. While valuable, they fail to show the spatial distribution of these errors. Here we discuss sources of model uncertainty, its typical measurement, and ways to depict it spatially. Our goal is to present practitioners with alternatives for providing more useful information to map consumers in the form of uncertainty maps.

Keywords: map uncertainty, model uncertainty, spatial analysis, spatial modeling, GIS modeling, remote sensing.

Overview of Map Error

Geospatial modelers using forestry data frequently make maps of model outputs. For example, maps of tree volume (Katila and Tomppo, 2002; Lister and Hoppus, 2002), land use and land cover (Vogelmann *et al.*, 2001), and species composition (Franklin *et al.*, 2000; Riemann Hershey *et al.*, 1997), have been produced using a wide range of statistical techniques. But it is rare to find a map of spatial uncertainty associated with these products. When examining these products, we are certain only that these maps contain both spatial and attribute uncertainty. Most maps are a representation of what occurs on the ground, so their accuracy is constrained by the accuracy and resolution of the information used to make them.

Spatial variability of map error can take many forms. For example, figures 1a-c illustrate georeferencing error that occurred between a satellite image (Landsat 5 image of CT) (http://edc.usgs.gov/products/satellite/tm.html) and a USGS 1:24000 digital line graph (http://edc.usgs.gov//products/map/dlg.html). Both the direction and magnitude of error, as represented by the arrows, vary spatially, probably due to spatial error in both data sources (figure 1d). With respect to modeled forestry data, biological conditions might differ at various locations on a mountain, and this may not be reflected in the remotely sensed reflectance data. Conversely, the areas may contain similar biological conditions but have substantially different reflectance values due to the topography. If topographic information is not incorporated into the model, errors may increase and the uncertainty likely will be greater. If map consumers are not provided with information on the data used in modeling, accuracy assessments specific to the mountains, or local (per pixel) uncertainties in the final map, they might be unaware that predictions associated with mountainous areas in this particular map generally are worse.

Errors in modeled maps originate from a variety of sources. Typically, geospatial modeling is performed by taking a set of known attribute data, referred to as reference or training data, and combining it with a set of geospatial predictor data like satellite imagery or GIS layers. The relationship between the attribute data (e.g., forest type, volume) and the predictor data is defined mathematically or used in another way to create a map. Errors occur in this context for a variety of reasons. There might be a weak functional relationship between the attribute of interest and the predictor data. Or, there might be a spatial mismatch between the predictors and the reference data. For example, GPS errors in ground data or georeferencing errors of satellite imagery contribute to this phenomenon.

The spatial mismatch might occur because of actual positional discrepancies or because the resolution of the predictor data does not correspond with that of the attribute of interest given local variation. For example, there might be as much variation in tree height within a 250-m pixel as there is across a 250-km^2 area. In this case, predictions of tree height at a given location would be poor.

Traditional Ways to Measure Error

Traditional methods for assessing map attribute errors are rooted in the literature on satellite remote sensing (Congalton *et al.*, 1983). For categorical maps, the usual accuracy assessment entails calculating an error matrix and kappa statistic. An error matrix is a tabular representation of the class specific and overall level of agreement between the attribute values found at known points and those found on the map. The kappa statistic is an index of the likelihood that the observed agreement between the known and mapped values occurred by chance (Congalton and Mead, 1983). These statistics and their derivatives form the basis of most published assessments of the accuracy of categorical maps.

For continuous maps, there are numerous methods found in the regression literature (Draper and Smith, 1981). The error structure is described by statistics such as root mean square error (the square root of the average squared deviation of each known value from each predicted value), the correlation coefficient and its regression analogue, the coefficient of determination (which are indices of the strength of the linear relationship between the known and predicted values), and other summaries of estimate and residual distributions.

Enhancements to the Traditional Ways

One problem with both of these approaches is that the error information is devoid of spatial information. For example, it is unclear what regions of the map have the highest and lowest errors. It also is unclear whether the errors are clustered in certain regions of the map or are distributed uniformly. Some map consumers might not care about map error. Others, such as scientists, managers, lawmakers, planners and map developers, might use the information to gain a greater understanding of the distribution of levels of the attribute across the landscape. The more information available to these users, the more effectively they can interpret, use, and iteratively improve the maps.

Different modeling techniques offer different options for providing spatial information on map error. One option is to produce an uncertainty map. This type of map attaches to each output pixel some measure of model reliability. In the case of figure 2, this measure is the average error of the regression models used to produce the estimates in the pixels.[1] Different models with different accuracies were used in different portions of the map, resulting in a map with spatially varying error. This general class of error map can be produced with most types of linear and many nonlinear modeling procedures.

Upon examining this map, one immediately has a greater understanding of the performance of the model. For example, potential users of the product from coastal New England might use the product more conservatively than someone in western Vermont. Similarly, scientists using the regional dataset for input to other models might consider a weighting scheme whereby the less reliable values carry less weight in the procedure.

Another option for depicting spatial error is to produce a map of a summary statistic from a set of simulations. Figure 3a is a map produced by a geostatistical procedure known as Sequential Gaussian Conditional Simulation (SGCS). In SGCS, multiple maps of estimates are produced to build at each location a distribution of possible values occurring at that location. These distributions are based on the configuration of the known data in space as well as the patterns in levels of the attribute to be predicted. Figure 3b shows a frequency histogram of estimates generated by SGCS at a single location. The 65th percentile is reported as the estimate and the interquartile range (IQR) is reported as the confidence index. Figure 3c shows the map of IQR associated with the map of estimates at each location (Riemann and Lister, 2004).

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[1] BLACKARD, J., FINCO, M., HELMER, E., HOLDEN, G., HOPPUS, M., JACOBS, D., LISTER, A., MOISEN, G., NELSON, M., RIEMANN, R., RUEFENACHT, B., SALAJANU, D., WEYERMANN, D., WINTERBERGER, K., BRANDEIS, T., CZAPLEWSKI, R., MCROBERTS, R., PATTERSON, P. and TYMCIO, R. *Mapping U.S. forest biomass using nationwide forest inventory data and MODIS-based information.* In preparation.

The SGCS approach has additional appeal because it highlights a fundamental principle of most spatial modeling exercises: we are nearly certain of a map's accuracy only at locations where something was measured on the ground. Between these locations, our modeling procedure fills in values that are in effect guesses based on the available known information. In other words, there are a wide range of possible values that might occur at a single unmeasured location, though some values are more probable given the configuration and values of the known data.

Other Strategies for Measuring and Analyzing Spatial Error

A multiscale analysis of the mean absolute difference between the average attribute value of the plots found in sets of grid cells superimposed over the area and the pixel-based estimates within those cells can be calculated for each of a number of spatial scales (figure 4) (Lister and Lister, in press). The goal of this type of analysis is to characterize the spatial agreement between the pixel-based estimates and the actual values with respect to area statistics, and also to reveal the scale(s) of agreement and of variation across the landscape. Plotting the average agreement between plots and pixels at each window centroid can reveal spatial patterns of discrepancies, possibly identifying regions of the map where the model broke down. Plotting the average window-based discrepancies across different spatial scales also can show the effect of scale of observation on the magnitude of discrepancies. For example, figure 5 indicates that for two modeling methods (a histogram matching procedure and one without the histogram matching), the pattern of decreasing dissimilarity (as measured by mean absolute error or MAE, or the average absolute difference between the biomass/ha of FIA plots and that of the pixels) follows that of the variance of the plots. Specifically, it is characterized by a sharp decline followed by a leveling off as the analysis window size increases. The shape of this relationship can help users gain an understanding of the agreement of model outputs and inputs at different spatial scales. For example, the level of agreement between training data and predictions at a scale of < 100,000 ha is much less than that at a scale of 500,000 ha. By understanding this relationship, users can better decide at what scale to interpret the data or use them in another model.

Conclusion

Errors are inherent in maps, especially ones created from geospatial modeling. Every map is a "best guess" of what actually occurs between locations on the ground that are measured. Most modeling procedures provide users with an indication of the level of confidence associated with the predictions generated. With a mapped index of confidence or accuracy, users gain a much deeper understanding of the quality of the map and its potential uses. With an error map, users can decide on tradeoffs they might make when using the map, for example, using a finer resolution but accepting more errors, or eliminating suspect areas of the mapped region from subsequent analysis. As advances occur in ecological geospatial modeling, it is critical that we incorporate spatial depictions of map error into our model outputs when possible.

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FIGURES

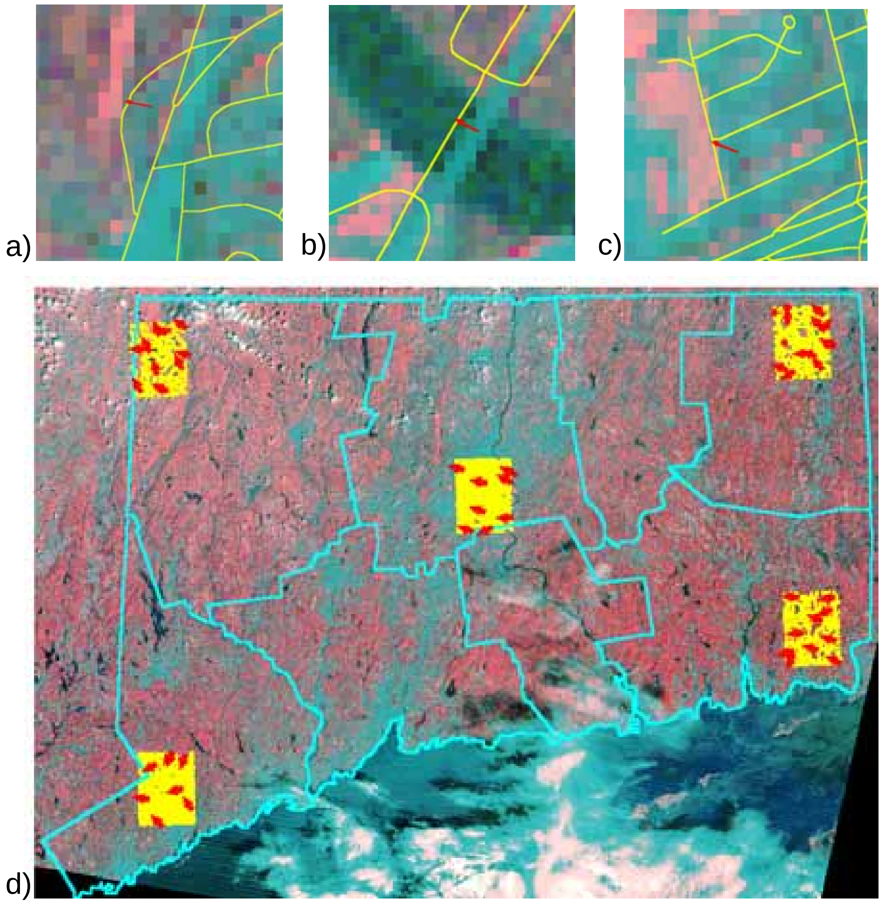

Figure 1.—USGS 1:24,000 digital line graph data overlaid on Landsat 5 Thematic Mapper imagery with arrows illustrating the direction and magnitude of the differences in geolocation. Figures a-c illustrate three specific examples; figure d depicts the differences within individual 1:24,000 quadrangles across Connecticut.

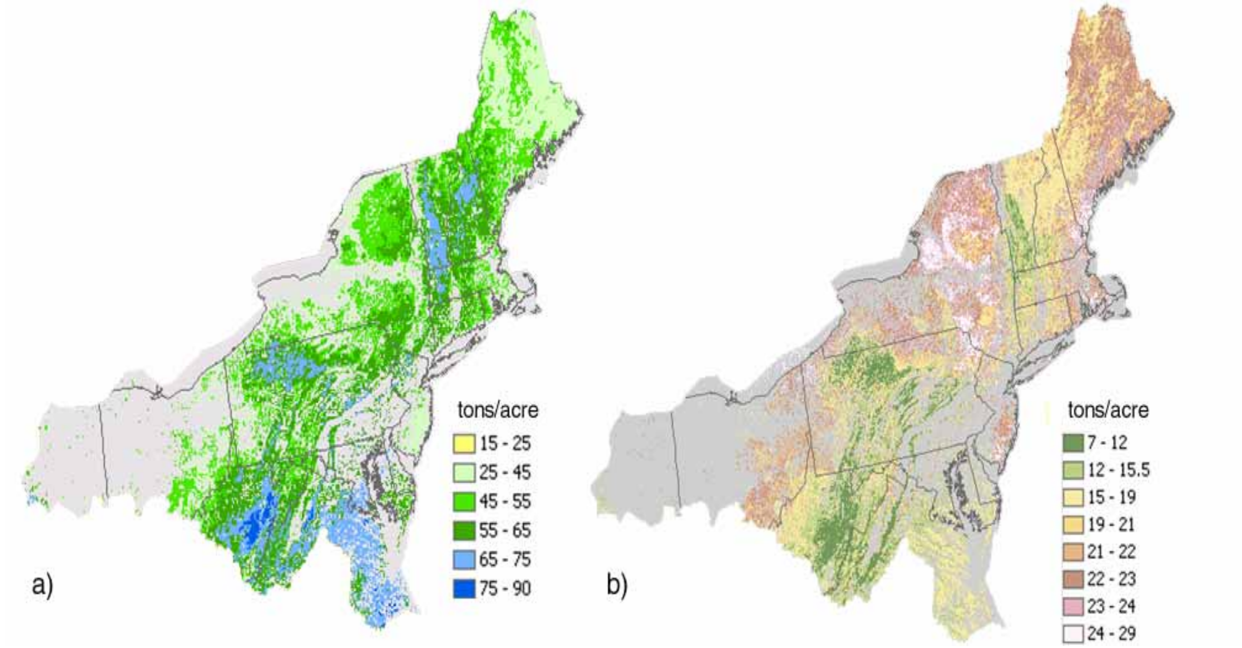

Figure 2.—a) A portion of a national biomass map (unpublished data) and b) associated uncertainty values for each modeled pixel.

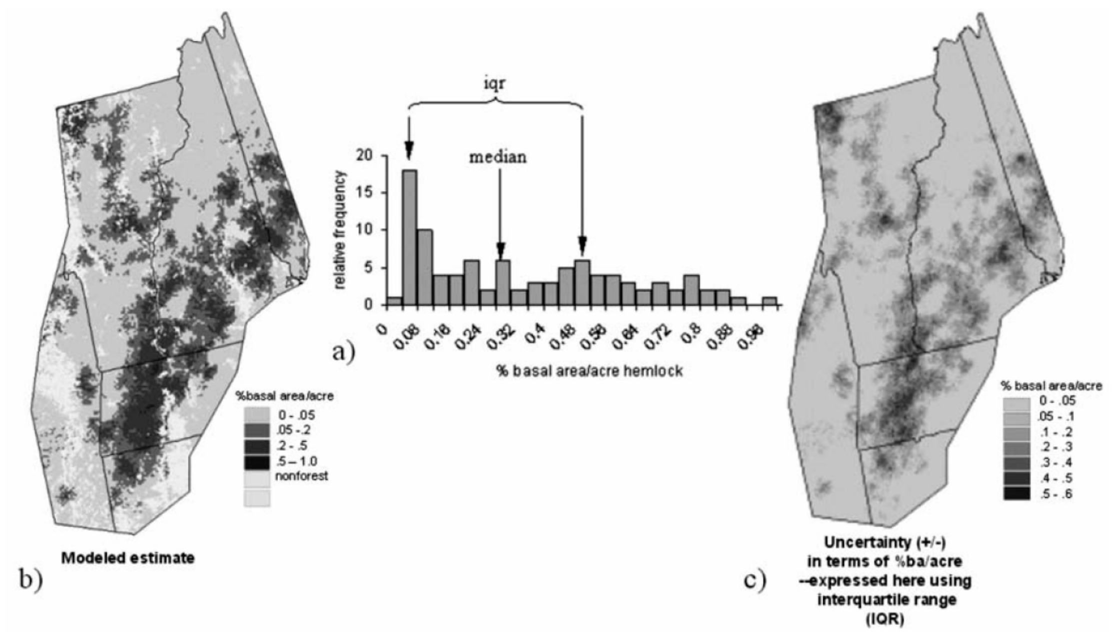

Figure 3.—SGCS creates a distribution of possible values for each pixel (one generated with each simulation) from which the user can easily extract a clear measure of the uncertainty of each local estimate: a) the distribution of values at a single, randomly chosen cell; b) the modeled estimate where the value at the 65th percentile was chosen for each pixel (with nonforest areas masked out); c) the value of the interquartile range (iqr) at each pixel, representing the range of uncertainty associated with each modeled value.

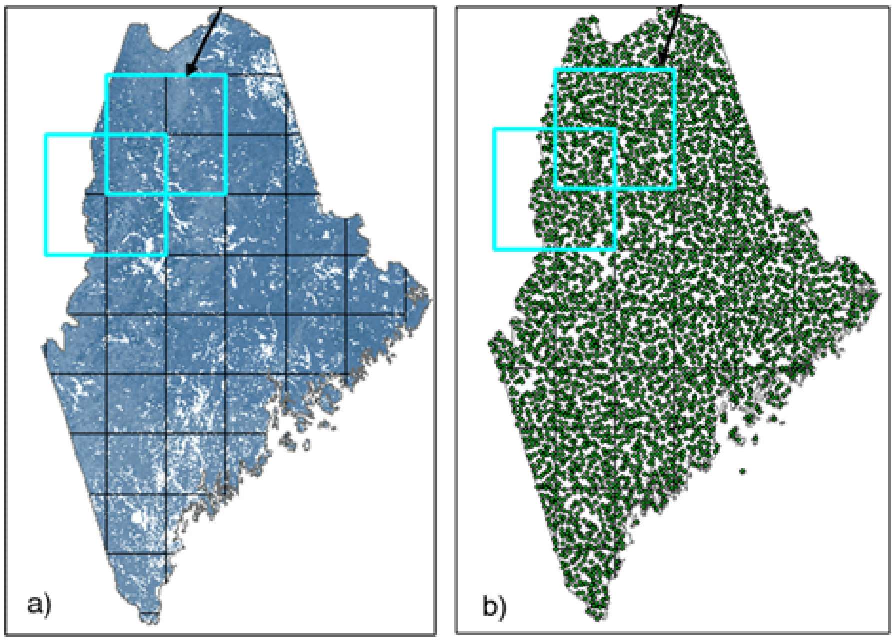

Figure 4.—Area summary statistics calculated from a modeled map (a) are compared with the same statistics (e.g. totals, means, and variances) calculated from the original plot attribute data (b). Boxes indicate overlapping analysis windows.

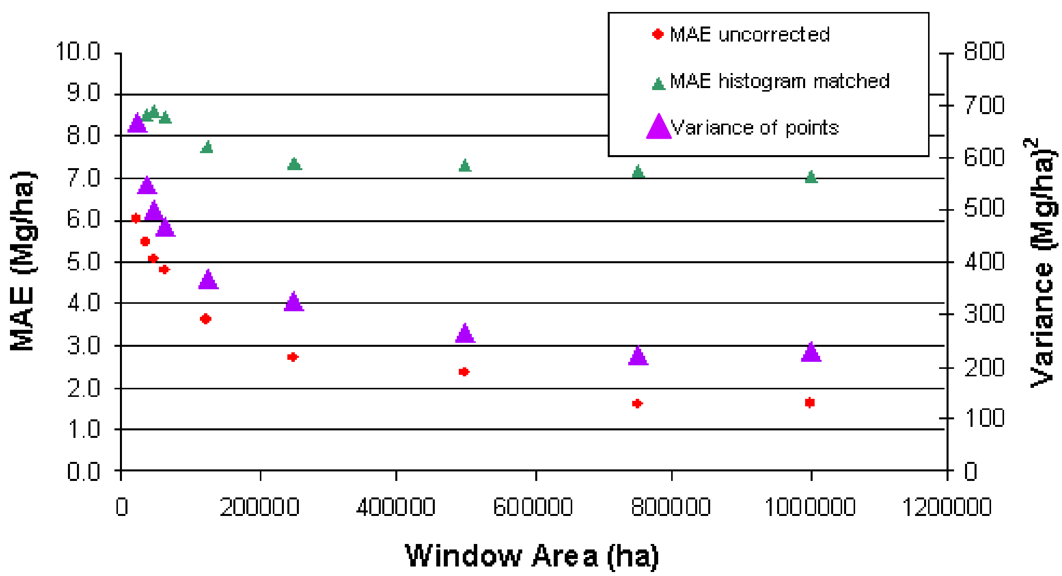

Figure 5. —Mean absolute error (MAE) --absolute value of average biomass/ha of FIA plots – average biomass/ha of pixels-- computed for the uncorrected and the histogram matched estimates at several scales. The variance of the window-based means of the plots for each window size also is shown (larger, purple triangles, secondary y axis).