The

Lignum Limited

RADIAL INCREMENT – NET GROWTH

Tree & Stand Growth Model

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Prepared For

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Introduction

The purpose of this paper is to describe the RADIAL INCREMENT – NET GROWTH or RING tree and stand growth model as it appears in the form of an algorithm. The basic equations used in the algorithm are described in Appendix 1.

RING Input Variables

The input variables are as follows.

Plot

Plot id number	A unique number representing the plot.
Plot label	A label that is commonly used to refer to the plot.
Plot est' year	The year when the plot was established.
Plot species class	The species class assigned on the basis of plot attributes.
Plot poly species class	The species class assigned on the basis of the inventory polygon attributes in which the plot resides.
Plot stand structure	The stand structure class to which the plot is assigned.

Tree

Plot id_number	
Tree id_number	A unique number representing the subject tree.
Tree number	A subject-tree number assigned at the time of plot establishment.
Tree species	The tree species.
Tree status	Alive or dead.
Tree dbh	Outside bark diameter (cm) at breast height (i.e. at 1.3 metres above the ground on the high side of the stump).
Tree height	A measured or calculated tree height (metres).
Tree age	A measured or estimated breast height tree age (years, i.e. age equals 1 when the tree has reached 1.3 metres).
Tree stems per hectare	The numbers of trees per hectare represented by the subject tree.

Input Assumptions

Maintain Table 1 as both an input open to modification and as a tabular summary available for reporting.

RING Input Assumptions

Table 1. Input assumptions.

Species		Plot	Species Assignments			Merchantability Limits				
Code	**Type**	**Summary (L,D,A,N)**	**Growth Equation**	**Height Equation**	**Kozak Volume Equation**	**Stump Height[1] (cm)**	**Min Dbh[2] (cm)**	**Min Top Diameter[1] (cm)**	**Log Length[2] (m)**	**Deductions Decay Waste Breakage (%)**
AC	H	N	FD	HS	Ac	30	12.5	10	3	5
AT	H	N	PL	AT	At	30	12.5	10	3	5
BL	C	L	FD	CS	B	30	12.5	10	3	5
DG	E	-	-	-	-	-	-	-	-	-
EP	H	N	PL	HS	E	30	12.5	10	3	5
FD	C	L	FD	FD	F	30	17.5	10	3	5
JR	E	-	-	-	-	-	-	-	-	-
PL	C	L	PL	PL	Pl	30	12.5	10	3	5
SX	C	L	FD	SX	S	30	17.5	10	3	5
VV	H	N	PL	HS	E	30	12.5	10	3	5
W	E	-	-	-	-	-	-	-	-	-
XC	E	-	-	-	-	-	-	-	-	-

[1] Integer. [2] Real, fixed to 1 decimal point.

Species Code: These are the codes used in the data input file. AC = Cottonwood. AT = Aspen. BL=Subalpine Fir. DG=Green Alder. EP=Paper Birch. FD=Dougflas-fir. JR=Juniper. PL=Lodgepole p\Pine. SX=Hybrid White-Englemann Spruce. VV=Cherry. W=Willow. XC=Other Conifer.

Species Type: These are the codes used in Table 3.1.6 below. H=Hardwood. C=Conifer. E=Exclude from data.

Species Assignment – Height Equation: For Douglas-fir (FD) see section 1.3. For Lodgepole Pine (PL) see section 2.3. For all other assignments see section 5.

Species Assignment – Volume Equations. See Table 3.6.1. in section 3.6.

Plot Summary: Include in the plot summary table statistics for either Live, Dead, or All Trees or Non of the Trees. Dead trees are reported.

In addition to the foregoing, the desired duration of the simulation must be set. Generally this will be less than 300 years, however it is recommended that 150 years be used initially.

RING Output Variables

<u>Tree</u>

Plot label	**String**
Calendar year	**Integer**. Staring from year of plot establishment; Note that if the plot was established in the first 4 months of the year, then the year of establishment is the previous year.
Tree number	**Integer**. Number assigned to subject tree in plot.
Tree status.	**String**. "Live" or "Dead".
Tree species	**String**. Actual tree species.
Dbh	**Real (Fixed at 1 decimal)**. Centemetres. Outside bark diameter
Trees per hectare.	**Real (Fixed at 1 decimal)**. Number of trees per hectare represented by subject trees in plot.
Basal area per hectare.	**Real (Fixed at 1 decimal)**. Metres squared per hectare represented by subject trees in plot (i.e. basal area per tree multiplied by trees per hectare for given subject tree).
Height.	**Real** (**Fixed at 1 decimal)**. Metres.
Live crown percent.	**Integer**. %. Left empty for now.
Taper class.	**Integer**. Left empty for now.
Volume_ws	**Real (Fixed at 1 decimal)**. Cubic metres per hectare. This is the whole stem volume of trees per hectare represented by a given subject tree (after multiplying by the trees per hectare that it represents) without any deductions for merchantability limits.
Volume_merch	**Real (Fixed at 1 decimal)**. Cubic metres per hectare. This is the volume per hectare represented by a given subject tree 9after multiplying by the trees per hectare that it represents) compiled to the merchantability limits identified in table
Tree bh age	**Integer**. Tree age in years at breast height.
Tree total age	**Integer**. Total tree age. Leave blank for now.

Note that with the exception of the tree age variables the kinds of variables are consistent with FPS output. Tree number replaces the "component" as assigned in FPS (FPS appends the component to the species name and records the results in a single field). In addition, FPS records some of the variables as integers where they are indicated as real numbers above.

Upon death, trees are reassigned a total age of 0. Tree age is then incremented with each year of simulation. This means that the number of records will increase with each year of simulation and tree numbers will be multiplied with respect to the dead tree component.

Plot

Plot label	**String**
Calendar year	**Integer**. Staring from year of plot establishment; Note that if the plot was established in the first 4 months of the year, then the year of establishment is the previous year.
QMD	**Real (Fixed at 1 decimal)**. Centemetres. Outside bark quadratic mean diameter, i.e. the basal area per hectare divided by the numbers of trees per hectare, Live tree component only.
Trees per hectare.	**Real (Fixed at 1 decimal)**. Total number per hectare.
Basal area per hectare.	**Real (Fixed at 1 decimal)**. Metres squared per hectare.
Dominant tree height.	**Real** (**Fixed at 1 decimal)**. Metres. Dominant tree height, this being equal to the average height of the 100 largest diameter (dbh) trees per hectare in the plot.
Lorey's mean height	**Real (Fixed at 1 decimal)**. Metres. This is the sum of the individual tree heights weighted by individual tree basal area per hectare (i.e. basal area per tree multiplied by trees per hectare represented by that tree in the plot).
Live crown percent.	**Integer**. %. Left empty for now.
Taper class.	**Integer**. Left empty for now.
Volume_ws	**Real (Fixed at 1 decimal)**. Cubic metres per hectare. This is the whole stem volume of trees without any deductions for merchantability limits.
Volume_merch	**Real (Fixed at 1 decimal)**. Cubic metres per hectare. This is the volume compiled to the merchantability limits identified in table
Dominant tree bh age	**Integer**. Years. Tree age in years at breast height. Dominant Tree total age **Integer**. Years. Total tree age including years to reach breast height. Leave blank for now.
Lorey's mean bh age	**Integer**. Years. The sum of the individual tree breast height ages weighted by individual tree basal area per hectare.
Lorey's mean total age	**Integer**. Years. The sum of the individual tree breast height ages weighted by individual tree basal area per hectare.

Note that with the exception of the tree age variables the kinds of variables are consistent with FPS output. Tree number replaces the "component" as assigned in FPS (FPS appends the component to the species name and records the results in a single field). In addition, FPS records some of the variables as integers where they are indicated as real numbers above.

Maintain Input Data

In addition maintain separate files for the plot and tree input data.

RING Algorithm

Individual Live Tree Growth & Mortality

Increment age

1. Increment breast height age to current age for each live subject tree.
2. Increment total age to current age for all dead trees and for live trees if total age > 0.

Tree and Stand Basal Area and Basal Area Increment

1. Compute $baph_{tree,t}$ (equations 4.1(1) and 4.2(1)) and sum the results ($baph_{stand,t,}$) for all living trees at simulation year t.
2. For each living tree in plot, given tree species and associated growth equation assignment (Table 1) compute basal area increment (bai_t) equations 1.1(1) or 2.1(1)).
3. Add basal area increment to basal area per tree and compute new tree dbh_{t+1} for year t+1 (equation 4.3(1)).
4. Repeat 1 to 4 for each time step and all living trees.

Height and Height Increment

1. Given tree species and associated species height equation assignment (Table 1) calculate height, h_{t+1} (equations 1.3(1) and 2.3(1)).

Survival and Mortality

1. For each tree age, where age equals, 1,2,3…maximum tree age given growth equation species (Table 1; equations 1.1(1) and 2.1(1)), calculate the total number of trees per hectare, TT_{age} (equations 1.2(1) and 1.2(2)) in a given single-year integer age class, regardless of species.
2. For each subject tree in the current (t+1) single-year integer age class and given TT_{age}, and diameter increment, din_{t+1} calculate the survival rate at time t+1, sr_{t+1} (equations 1.2(1), 1.2(2)).
3. Calculate the number of trees per hectare that survive with respect to each subject tree at time t+1 ($tph_{t=0} * sr_{t+1}$).
4. Calculate the number of trees per hectare that die with respect to each tree at time t+1 ($tph_{t=0} * (1-sr_{t+1})$). Assign a total age of 1.

Whole Stem Volume

1. Do this for living subject trees only and only if subject tree height >= minimum stump height plus log length (Table 1) for given species.
2. Set merchantable stem volume (*Volume_ws*) to 0.

3. Set height to minimum stump height (in metres from Table 1, e.g. $h = 0.3$ for a minimum stump height of 30 cm), and calculate relative height, Z (equation 3.3(1)) and X (equation 3.4(1)).
4. Compute inside bark diameter at breast height, d_{butt} (equation 3.2(1)).
5. Compute inside bark diameter at butt end of log (equation 3.1(1)).
6. Set log volume equal to 0.
7. Increment height, h by minimum log length for species (Table 1).
8. If height, h is less than total tree height, H then calculate Z, X, and top end diameter, d_{top}; Otherwise log volume equals 0, go to 13.
9. Calculate log volume using top and bottom end radii ($d_{top}/2$ and $d_{butt}/2$) and equation 4.4(1).
10. Add log volume to whole stem volume.
11. Set $d_{butt} = d_{top}$.
12. Go to step 6.
13. Multiply whole stem volume by the live number trees per hectare for given subject tree.
14. Multiply whole stem volume by one minus the proportion lost due to decay, waste and breakage (Table 1).

Merchantable Volume

1. Do this for living subject trees only and only if whole tree volume is greater than 0.
2. Set whole stem volume (*Volume_merch*) to 0.
3. Calculate merchantable tree height: Set d_i equal to minimum top diameter given species (Table 1). Set relative tree height, Z_0 equal to 0.9.
4. Calculate C (equation 3.5(2), where $D = dbh$ using Z_0). Calculate the second estimation of relative height Z_1 using equation 3.5(1).
5. If condition 3.5(4) is met then apply 3.5(5) to calculate merchantable tree height, H_{merch} and go to step 6; Otherwise let $Z_0 = Z_1$ and go to step 4.
6. Set height, h equal to minimum stump height (Table 1, e.g. $h = 0.30$ for a minimum stump height of 30 cm) and calculate Z (equation 3.3(1)) and X (equation 3.4(1)).
7. Compute inside bark diameter at breast height, d_{butt} (equation 3.2(1)).
8. Compute inside bark diameter at butt end of log (equation 3.1(1)).
9. Set log volume equal to 0.
10. Increment height, h by minimum log length for species (Table 1).
11. If height, h is less than merchantable tree height, H_{merch} then calculate Z, X, and top end diameter, d_{top}; Otherwise log volume equals 0, go to 16.
12. Calculate log volume using top and bottom end radii ($d_{top}/2$ and $d_{butt}/2$) and equation 4.4(1).
13. Add log volume to whole stem volume.
14. Set $d_{butt} = d_{top}$.
15. Go to step 9.

16. Multiply whole stem volume by the live number trees per hectare for given subject tree.
17. Multiply whole stem volume by one minus the proportion lost due to decay, waste and breakage (Table 1).

Update Subject Tree List

1. For all live subject trees that existed in the previous time step (t=0) add them to the record and update output variables for current time step.
2. For all dead subject trees that existed in the previous time step (t=0) add them to the record along with the attributes from the previous time step. Update age to, t+1.
3. For all dead subject trees that were created in the current time step (t+1), add them to the subject tree list, set total tree age = to 1, set tree status = "Dead", record dbh_{t+1}, the number of dead trees per hectare, $tph_{dead,t+1}$, tree height, h_{t+1}, Volume_ws, Volume_merch

Appendix I
Growth & Tree Volume / Taper Equations

1. Douglas-fir (FD) Growth Equations

1.1. Individual Tree Basal Area Increment

$$bai = 62.231 * EXP(-0.030 * BAL - 0.023 * BAS) * (1 - EXP[-0.003 * dbh^2 * EXP[-0.009 * age]]) \quad 1.1(1)$$

where

bai is the outside bark basal area increment per tree for a given subject tree (cm^2).
BAL is total stand basal area per hectare greater than or equal to the dbh of the subject tree (m^2).
BAS is total stand basal area per hectare less than the dbh of the subject tree(m^2).
dbh is the subject tree outside bark diameter at breast height (1.3 metres; cm).
age is the subject tree age at breast height (y).

1.2. Individual Tree Survival

$$sr = \left(\frac{1}{TT_{age}}\right)^{\left(\frac{1}{age_{\max} - age}\right)} - \left[\frac{1}{1 + EXP(21.9196 * din + dbh^{0.4975})}\right] \quad 1.2(1)$$

where

sr is the survival rate of the subject tree, where $0 \leq sr \geq 1$.
TT_{age} is the total number of trees, regardless of species, where age = 1,2,3 … age_{max} (in integers), and such that age is equal to the age (y) of the subject tree.
age_{max} is the maximum age (y) for a given species, in this case interior Douglas-fir = 567 years.
age is the age of the subject tree (y, integers only).
din is the estimated outside bark diameter increment for 1 year (cm), derived from subject tree dbh and bai from equation 1.

1.3. Tree Height

$$H_{t+1} = 1.3 + 39.772 * (1 - EXP(0.024 * dbh_{t+1})) * (1 - EXP(0.058 * age_{t+1})) * \left(\frac{H_{t=0}}{pH_{t=0}} \right) \quad 1.3(1)$$

where

ht_{t+1} is the height (m) at the next time step given dbh at the next time step.
dbh_{t+1} is the outside bark diameter(cm) estimated using diameter at the previous time step, plus the increment in basal area from equation 1.
age_{t+1} is breast height age after the dbh increment.
$H_{t=0}$ is the initial tree height (m) at time t=0.
$pH_{t=0}$ is the predicted tree height (m) at time t=0 given $dbh_{t=0}$ using that portion of equation 3 included in the large square parentheses.

2. Lodgepole Pine (PL) Growth Equations

2.1. Individual Tree Basal Area Increment

$$bai = 15.522 * EXP(-0.017 * BAL) * \left(1 - EXP\left[-0.004 * dbh^2 * EXP[-0.008 * age]\right]\right)$$

2.1(1)

2.2. Individual Tree Survival

$$sr = \left(\frac{1}{TT_{age}}\right)^{\left(\frac{1}{age_{\max} - age}\right)} - \left[\frac{1}{1 + EXP\left(34.2408 * din + 0.2468 * dbh - 0.0054 * dbh^2\right)}\right]$$

2.2(1)

where

age_{max} is equal to 332 years for lodgepole pine.

2.3. Tree Height

$$H_{t+1} = 1.3 + 34.252 * (1 - EXP(0.036 * dbh_{t+1})) * (1 - EXP(0.051 * age_{t+1})) * \left(\frac{H_{t=0}}{pH_{t=0}}\right)$$

2.3(1)

3. Individual Tree Volume[1] / Taper Equations

3.1. Tree Diameter for a Given Height

$$\frac{d_i}{DI} = X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3 Z^{0.5} + b_4 e^Z + b_5 (D/H)}$$ 3.1(1)

where

d_i	is diameter inside bark (cm) for a given relative height, Z.
DI	is inside bark diameter at breast height, (cm, i.e. dbh_{ob}; equation 3.1(2)).
D	is the outside bark diameter at breast height, (cm, i.e. dbh_{ib})
H	is the total height (m) of the tree.
Z	see equation 3.1.3(1).
X	see equation 3.1.4(1).
e	is the exponent, 2.718…
a_x, b_x	are parameters.

3.2. Diameter Inside Bark at Breast Height

$$DI = a_0 D^{a_1} a_2^{\,D}$$ 3.2(1)

3.3. Relative Tree Height

$$Z = \frac{h}{H}$$ 3.3(1)

where

h is the height from the base of the tree(m), such that 0≤ h ≥ H.

3.4. Adjusted Relative Tree Height

$$X = \frac{\left[1 - \left(\frac{h}{H}\right)^{0.5}\right]}{1 - p^{0.5}}$$ 3.4(1)

where

[1] From: Kozak, A. 1988. A variable exponent taper equation. Can. J. For. 18:1363-1368.

$$p = \frac{h}{H} \tag{3.4(2)}$$

at the point where

$$d_i = DI \tag{3.4(3)}$$

3.5. <u>Merchantable height calculation</u>

$$\frac{h_i}{H} = \left[1 - \left(\frac{d_i}{a_0 D^{a_1} a_2^D} \right)^{\frac{1}{C}} \left(1 - p^{0.5}\right)^2 \right] \tag{3.5(1)}$$

where

$$C = b_1 \left(\frac{h_i}{H} \right) + b_2 \ln\left(\frac{h_i}{H} + 0.001 \right) + b_3 \left(\frac{h_i}{H} \right) + b_4 e^{\left(\frac{h_i}{H} \right)} + b_5 \left(\frac{D}{H} \right) \tag{3.5(2)}$$

In equation 3.1.5(1) d_i is set to the desired top diameter (e.g. 10cm top diameter) and C is calculated from a guessed value of h_i/H, termed $(h_i/H)_0$. A good starting point for $(h_i/H)_0$ is 0.9. Once C is calculated using the guessed value, the first estimation $(h_i/H)_1$ can be obtained from equation 3.1.5(1). Having calculated $(h_i/H)_1$, the next value used to estimate C can be obtained by:

$$\left(\frac{h_i}{H} \right)_2 = \left(\frac{h_i}{H} \right)_0 + \left[\frac{\left(h_i/H \right)_1 - \left(h_i/H \right)_2}{2} \right] \tag{3.5(3)}$$

The above procedure is repeated until a certain precision is obtained, e.g.

$$\left| \left(\frac{h_i}{H} \right)_j - \left(\frac{h_i}{H} \right)_{j+1} \right| < 0.01 \tag{3.5(4)}$$

For most trees, two to five cycles will result in 1%(0.01) precision in the merchantable height calculation. Once the desired precision is obtained, the merchantable height (h_m, m) can be calculated:

$$h_m = H\left(\frac{h_i}{H}\right)_j \qquad 3.5(5)$$

3.6 Kozak Variable Taper Equation Parameters

FIZ = D,E,F,G,H,I,J

Table 3.6.1. Parameters for Kozak Variable Taper Equations by tree species[2].

	Parameters								
Species[1]	a_0	a_1	a_2	b_1	b_2	b_3	b_4	b_5	p
F	0.920840	0.923867	1.000568	1.095560	-0.202191	0.967329	-0.514604	0.081697	0.25
C	1.106789	0.864250	1.000646	1.419220	-0.324810	1.485930	-0.823428	0.103945	0.30
H	0.783564	1.009780	1.000107	0.782293	-0.126143	0.865118	-0.313180	0.054607	0.25
B	1.008741	0.916357	1.001159	1.415990	-0.325671	2.793270	-1.326790	0.108427	0.30
S	0.897311	0.957090	0.999370	1.532270	-0.364679	2.741210	-1.362760	0.117756	0.30
Y	0.928138	0.945293	0.999206	0.301423	-0.040792	-1.235630	0.672879	0.030743	0.30
PW	0.984019	0.941322	0.999700	1.571030	-0.369344	2.703200	-1.334700	0.049628	0.25
PL	0.775601	1.040320	0.996984	0.745750	-0.130177	0.558818	-0.324178	0.198687	0.25
PY	0.856592	0.936402	1.002104	0.566217	-0.087141	-0.063450	0.051415	0.071720	0.25
L	0.746827	1.003900	0.997233	0.747048	-0.133729	0.397110	-0.183542	0.078345	0.30
AC	0.802839	0.993776	0.998974	0.706093	-0.096789	0.312724	-0.080057	0.119634	0.25
D	0.719188	1.052190	0.997551	0.599235	-0.033036	-0.261339	0.123059	0.215536	0.30
MB	1.097880	0.840504	1.006569	0.981297	-0.222619	1.428220	-0.654456	0.228560	0.25
E	0.648830	1.121390	0.992077	0.865974	-0.106757	0.257139	-0.149926	0.254574	0.25
AT	0.855966	0.987014	0.999828	0.424473	-0.037553	-0.517540	0.303931	0.102211	0.20
PA	1.078961	0.894083	1.001749	1.377540	-0.286807	1.038780	-0.647372	0.072537	0.25

F=Douglas-fir, C=Western Red Cedar, H=Western Hemlock, B=Sub-alpine Fir, S=Spruce spp., Y=Yellow Cedar, Pw=Western White Pine, Pl=lodgepole pine, Py= Ponderosa Pine, L=Larch AC=Cottonwood, D=Alder, Mb=Bigleaf Maple, E=Birch, At=Aspen, Pa=Whitebark Pine.

[2] Parameters derived from computer code: <\SSD\MACROS\FUNCTIONS\VOL_KOZ3.SAS> Migrated on April 28, 1997 from Ministry of Forests, Victoria, B.C.

4. Basic Equations

4.1. Basal Area Per Tree

$$ba = 3.141592654 * \left(\frac{dbh}{200}\right)^2 \qquad 4.1(1)$$

where

ba is basal area per tree (m^2).
dbh is tree diameter (cm) at breast height (1.3 metres).

4.2. Tree Basal Area Per Hectare

$$baph = ba * tph \qquad 4.2(1)$$

where

baph is the basal area per hectare (m^2ha^{-1}) represented in a plot by a given subject tree.
tph is the number of trees per hectare represented in the plot by a given subject tree (# ha^{-1}).

4.3. Tree Diameter

$$dbh_{t+1} = \left(\frac{ba + bai}{3.141592654}\right)^{0.5} * 200 \qquad 4.3(1)$$

where

bai is the basal area increment estimated for a given subject tree (m^2) derived from the growth equations: 1.1(1), 2.1(1).

4.4. Tree Diameter Increment

$$din = dbh_{t+1} - dbh_{t=0} \qquad 4.4(1)$$

where

din is tree outside bark diameter increment (cm).
dbh_{t+1} is the outside bark diameter (cm) at time, t+1 after being incremented.
$dbh_{t=0}$ is the outside bark diameter (cm) before being incremented at time, t=0.

4.5. Log Volume – Smalian's Formula

$$V_{\log} = \left(r_{top}^2 + r_{bottom}^2\right) * L * 0.0001570796 \qquad 4.5(1)$$

where

V_{log} is the volume of the log (m^3).
r_{top} is the radius of the top end of the log (cm).
r_{bottom} is the radius of the bottom end of the log (cm).
L is the length of the log (m).

4.6. Lorey's Mean Statistics

$$S = \frac{\sum_{i=1}^{n} X_i baph_i}{\sum_{i=1}^{n} baph_i} \qquad 4.6(1)$$

where

S is Lorey's mean stand statistic (height or age).
X_i is the statistic for subject tree "i", where $1 \le i \le n$, and there are "n" (alive) trees in the plot.
$baph_i$ is the basal area per hectare represented by subject tree "i".

5. Height Equations – Other Species

For variable definitions see section 1.3. The Douglas-fir height equation (FD) is described in section 1.3. The lodgepole pine (PL) is described in section 2.3.

5.1. Aspen – AT

$$H_{t+1} = 1.3 + \left[\frac{29.47551}{1 + \left(\frac{dbh_{t+1}^{-1.297234}}{0.03027} \right)} \right] * \left(\frac{H_{t=0}}{pH_{t=0}} \right) \qquad 5.1(1)$$

5.2. Conifer Species – CS

$$H_{t+1} = 1.3 + \frac{dbh_{t+1}^{2}}{3.966253 + 0.193568 * dbh_{t+1} + 0.036821 * dbh_{t+1}^{2}} * \left(\frac{H_{t=0}}{pH_{t=0}} \right) \qquad 5.2(1)$$

5.3. Hardwood Species – HS

$$H_{t+1} = 1.3 + 20.5038 * \left(1 - EXP\left(-0.015282 * dbh_{t+1}^{1.737736}\right)\right) * \left(\frac{H_{t=0}}{pH_{t=0}} \right) \qquad 5.3(1)$$

5.4. Spruce – SX

$$H_{t+1} = 1.3 + 35.96108 * \left(1 - EXP\left(-0.039653 * dbh_{t+1}\right)\right)^{1.395898} ** \left(\frac{H_{t=0}}{pH_{t=0}} \right) \qquad 5.4(1)$$