Possible Approaches for Relating Growth and Yield Models to Aerial Information

PART II – Example Using Existing Audit Data

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Executive Summary

Expressing existing stand conditions using a tree list is useful for landscape level analysis and planning purposes. Many growth and yield (GY) models and simulators require some form of tree list to set initial conditions for the stand being modeled or projected. This project was conducted to illustrate how tree lists can be generated from aerial information. The methods used and tree lists generated using parameter prediction and the Most Similar Neighbor (MSN) approaches are presented. Based on one example data set, recommendations are:

1. Combined use of the MSN and the parameter prediction approaches is suggested for generating tree list from aerial information.

2. A detailed documentation of the MSN fitting and parameter prediction approach for use with BC data sets is needed.

3. Larger data sets should be used for generating tree lists from aerial information. Audit polygons for two or more TSAs can be combined.

4. Further testing of both the parameter prediction and the MSN approaches is needed. Testing should include altering the ground (X set) and aerial (Y set) variables for the MSN approach, and altering fitting techniques and model forms for the parameter prediction approach.

5. Generating tree lists by biogeoclimatic (BEC) zone rather than by timber supply area (TSA) and by species group or species guild rather than by individual tree species might result in better estimates.

6. Additional ground variables and aerial information (e.g., landsat imagery) should be obtained and the advantages of using these additional variables should be assessed.

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Acknowledgments

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List of Species Codes Used

AT	Aspen (*Populus tremuloides*)
AC	Cottonwood (*populus balsamifera*)
B	True fir (*Abies* spp.)
BL	Subalpine fir (*Abies lasiocarpa*)
E	Birch (*Betula* spp.)
CW	Western red cedar (*Thuja plicata*)
FD	Douglas-fir (*Pseudotsuga menziesii*)
HM	Mountain hemlock (*Tsuga mertensiana*)
HW	Western hemlock (*Tsuga heterophylla*)
L	Larch (*Larch* spp.)
LW	Western larch (*Larix occidentalis*)
PA	Whitebark pine (*Pinus albicalis*)
PL	Lodgepole pine (*Pinus contorta*)
PW	Western white pine (*Pinus monticola*)
PY	Ponderosa pine (*Pinus ponderosa*)
S	Spruce (*Picea* spp.)
SE	Englemann spruce (*Picea engelmanii*)

1. INTRODUCTION

1.1. Background

The province of British Columbia (BC) manages 59 million ha (145.8 million acres) of forestland similar in area to the entire national forests of the US (77.3 million ha, 191 million acres). This forestland is divided into strata (polygons) based on forest cover types for forest inventory purposes. Each of these polygons has aerial attributes such as species composition (in % of crown closure), crown closure (%), height class, age class, site class/site index, elevation, biogeoclimatic (BEC) zone/subzone, and stocking class. Additional variables derived from the Variable Density Yield Projection (VDYP) model include volume/ha, and quadratic mean diameter.

The province of BC has also developed several growth and yield models (GY) for use in various regions of the province. Among these models, Prognosis^{BC} (adapted from the northern Idaho variant) is used in the Southeastern part of the province, while mixed-wood growth model (MGM) is used in Northeast part of the province. However, the estimates from these models have not been linked to aerial stand attributes or to the existing inventory database, and they have not been used to update polygon level estimates.

A tree-list is a tree-by-tree record of the tree species, diameter measures at breast height (dbh; measured at 1.3 m above ground), height, and stems per ha, while a diameter distribution indicates the frequencies of trees within defined diameter intervals (Clutter et al. 1983). Knowledge of the tree-list or the diameter distribution within a stand is useful in planning and designing new processing facilities, in supplying and operating existing processing facilities, in assessing stand structure and in assessing biodiversity at stand level. Expressing existing stand conditions using a tree list is also useful for landscape level analysis and planning purposes. Many GY models and simulators require some form of tree list to set initial conditions for the stand being modeled or projected. In such growth models, the use of an existing (actual) tree list to define an initial stand condition provides reasonable growth and yield estimates and future stand conditions. However, in many situations, individual tree measurement data from which a tree list may be constructed are not available. Hence, the ability to generate a reasonable tree list from a minimal amount of information or from an existing information source such as forest cover labels is invaluable.

1.2. Objectives

The overall objectives of this project were to:

1) review the methods that are used to relate growth and yield models to aerial information;
2) synthesize some of the methods in (1) in light of the available data sources in the Province;
3) assess selected methods for accuracy and suitability for the Resources Inventory Branch of the Ministry of Forests; and
4) provide recommendations based on the findings of (1), (2), and (3).

To achieve these objectives, first a report including a literature review of approaches that are used to generate tree lists from aerial information, a listing of possible data sources for BC, and a discussion on the challenges (opportunities) in relating in relating GY models to aerial information and inventory databases was prepared (Temesgen and LeMay 2000a). Second, a one-day tree list generation workshop (TLGW) was held on January 21, 2000 at the University of British Columbia to discuss methods and ideas on a generating tree list from aerial information, to link GY model to the land base. Twenty-one model developers and forest biometricians participated in this workshop. Discussions at this workshop were summarized and submitted to the MOF and to the participants of the workshop (Temesgen and LeMay 2000b). The TLGW participants agreed that the audit data sets could be used to assess the performance various approaches to generate tree lists, and that linking more than one approach might give better results.

Based on the literature review and TLGW, a parameter prediction and the Most Similar Neighbor (MSN) approaches to generating tree lists were selected and illustrated using a sample of existing audit data. The results of this illustration are presented in this report. Also recommendations are given for generating tree lists from aerial data, and to link GY models to the land base.

2. METHODS

2.1. Data Preparation

The Arrow timber supply area (TSA) audit data set for mature stands was provided by Ministry of Forests, Resources Inventory Branch in order to evaluate potential tree list generation approaches. The Arrow TSA is located in the southeast interior of B.C. and covers the interior cedar-hemlock (ICH), Englemann-Spruce Subalpine Fir (ESSF) and pockets of Alpine Tundra bio geoclimatic (BEC) zones. For the audit sample, 50 polygons were selected with probability proportional to polygon size (PPS). In each sampled polygon, a point was randomly selected from a grid, and nine plots were located at the selected grid point.

The data set was prepared for further analysis as follow:

1. Dead trees (i.e., tree classes 4 and 6) were excluded. Due to the limited number of hardwoods in the audit sample plots, hardwoods were combined into one group ("MH'').

2. Since the audit data set was collected using variable radius plots (VRPs), the expansion factor (TF) was calculated for each tree. Stand level attributes such as basal area and stems per hectare were computed for each species and diameter class of each plot. The plots of each cluster were combined by averaging all variables.

3. For each sampled polygon, associated Forest Inventory Projection (FIP) data were obtained and aerial attributes were extracted. The aerial attributes included: species and their respective percent composition, site index in m, age class, height class, crown closure (%), BEC zone, site series, stand age (years), and projected height (m). The volume (m^3/ha) and quadratic mean diameter (cm), for all trees above 12.5 cm diameter outside bark at breast height (DBH; 1.3 m above ground) based on VDYP (BC Ministry of Forests 1995) projections were also extracted.

4. Midpoints of all class data were used to represent the class. Trees in height classes 2, 3, and 4 were assigned a height of 15, 25, and 35 metres, respectively. Trees in age classes 4, 5,6,7,8 and 9 were assigned an age of 70, 90, 110, 130, 195, and 265 years, respectively.

Using these data, the parameter prediction and the Most Similar Neighbor (MSN) tree lists generation approaches were assessed.

2.2. Parameter Prediction

For each species and polygon combination, the proportion of trees (*f(x)*, frequency) represented by a given DBH was calculated by dividing the average number of trees represented by each tallied tree by the total number of trees of that species. These proportions were cumulated by DBH to calculate the cumulative frequency *(F(X))*. Following this, the Weibull probability density function (PDF) and cumulative density function (CDF) were fitted. The Weibull PDF is (Clutter *et al*. 1983):

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} \exp\left\{-\left(\frac{x-a}{b}\right)^{c}\right\} \qquad \text{for x} \geq \text{a, a} \geq 0\text{, b, c} > 0$$
$$= 0\text{, elsewhere}$$

Where *x* is *DBH*, *exp* is the Naperian constant, *a*, *b*, and *c* are location, scale, and shape parameters, respectively.

The Weibull CDF is (Clutter *et al*. 1983):

$$F(X) = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \quad \text{for x} \geq \text{a, a} \geq 0\text{, b, c} > 0$$
$$= 0\text{, elsewhere}$$

Percentile-based parameter estimation equations were used to obtain starting values for each polygon and species combination. The equation for *a* is (Zanakis 1979):

$$\hat{a} = \frac{x_1 x_n - x_2^{\;2}}{x_1 + x_n - 2x_2} \quad \text{if } \mathrm{x}_2 \text{ is closer than } \mathrm{x}_1 \text{ than to } \mathrm{x}_\mathrm{n}\text{, and } \mathrm{x}_1 \text{ otherwise.}$$

Where x_i is the i^{th} ordered *DBH* value (from smallest to largest) in the sample and *n* is sample size. If *a* is less than zero, it is set equal to x_1 (i.e., the minimum DBH). The scale parameter is estimated as:

$$\hat{b} = -\hat{a} + x_{[0.63n]}$$

Where *0.63n* represents the 63^{rd} percentile of the cumulative frequency by DBH, and *[ ]* indicates rounding up to the nearest ordered DBH value. The shape parameter is estimated as:

$$\hat{c} = \frac{\ln\left[\frac{\ln(1-p_k)}{\ln(1-p_i)}\right]}{\ln\left[\frac{x_{[npk]}-\hat{a}}{x_{[npi]}-\hat{a}}\right]}$$

Where p_i is the 0.16731 percentile and p_k is the 0.97366 percentile. Inputting these starting values, the Marquardt optimization technique, (SAS Institute Inc. 1989) was used to estimate Weibull parameters. When the solution failed to converge, different starting values were used to obtain stable parameter estimates for the Weibull distribution.

To obtain stable parameter estimates, only polygons with more than four tallied trees of a species were fitted (154 species and polygon combinations). After parameters were estimated, they were regressed against aerial attributes using multiple linear regression by species. The number of trees per hectare, all species, was also estimated as a function of aerial attributes. The error in predicting stems per hectare was summarized into two fit statistics:

1. The I-squared value

$$I^2 = 1 - \frac{SSE}{SSY}$$

Where SSE is sum squares of error and SSY is sum squares of total over all 50 polygons. The I^2 is similar to R^2 value calculated for multiple linear regression.

2. Estimated root mean square (ERMSE)

$$ERMSE = \sqrt{\frac{SSE}{(n-3)}}$$

Where n is 50 polygons.

To obtain the estimated number of stems by species and the DBH class:

1) the stems per hectare were estimated using aerial data;
2) the estimated number of trees per hectare (ESPH) was prorated based on aerial species composition (as specified on the forest cover map);

3) the Weibull parameters were predicted from stand aerial attributes for each species, and the proportion of trees in each DBH class was calculated; and
4) the frequency by DBH class was calculated by multiplying the estimated stems per hectare for the species by the respective proportion.

The observed and predicted frequencies by species, and then by species and DBH, were compared.

2.3. Most Similar Neighbor Method (MSN)

The 50 sample polygons were randomly divided into reference (80%) and target (20%) polygons. The “reference polygon” are sampled polygons that have both ground inventory and aerial attributes, while “target polygons” refers to un-sampled polygons that only have aerial attributes data. Reference polygons constitute the pool of potential most similar neighbors that can be selected to impute the tree list on to target polygons. They are used to develop similarity function in selecting a neighbor stand in MSN.

In this analysis, the 20% target polygons were assumed to be unsampled polygons (missing ground data), and were used to validate the accuracy of the MSN approach by comparing the observed tree list for the polygon to the expected tree list obtained by substituting the selected reference polygon. The MSN software provided by Dr. Melinda Moeur of USDA was used. The MSN involves two steps to link a reference stand to a target stand. First, canonical correlation between ground (Y set) and aerial data (X set) is used to determine weights to be applied to the aerial data. Second, the “most similar” sampled polygon is selected based on the aerial data, weighted by the correlations to the ground data, and linked to a target stand (Moeur 1995).

To limit the number of independent variables in the MSN analysis, the basal area per hectare was partitioned into four species groups based on shade tolerance. The following 13 ground inventory variables were selected for the MSN analysis (Y set):

1. BA_CW, representing basal area of CW, HW, and S in m^2/ha
2. BA_PL, representing basal area of PA, PL, PW, and PY in m^2/ha
3. BA_FD representing basal area of BL and FD in m^2/ha
4. BA_MH representing basal area of AC, AT, E, and L in m^2/ha
5. Minimum (MIN) diameter in cm
6. Maximum (MAX) diameter in cm

7. The 16.7 (ONE16) percentile for the stems/ha by DBH
8. The 63 (ONE63) percentile for the stems/ha by DBH
9. The 97 (ONE97) percentile for the stems/ha by DBH
10. Basal area (BA) per hectare in m^2/ha
11. Number of trees per hectare (SPH)
12. BEC zones (3) represented by two dummy variables

The following 23 aerial stand variables were selected for MSN analysis (X set) (SPCSx indicates species composition obtained from the FIP file):

1. First leading species % composition (PCT1)
2. Second leading species % composition (PCT2)
3. Height class (HTG_CL) midpoint in m
4. Age class (AGE_CL) midpoint in years
5. Site index (SITE_IDX) in m
6. Crown closure (CC) in %
7. Projected stand height (HT_PRO) in m. HT_PROJ refers to the height of the stand, as measured or estimated at the reference year
8. Volume (VOLT) in m^3/ha
9. Quadratic mean diameter (QMD) in cm
10. First leading species represented by seven dummy variables
11. Second leading species represented by seven dummy variables

After the most similar neighbor (a reference stand) was selected for each target stand, the DBH class frequency, and ground and aerial variables of the target and the selected reference polygons were compared. Stand tables of the target and the selected reference polygons were plotted by species for comparison.

3. RESULTS AND DISCUSSION

3.1. Data Summary

The Arrow TSA audit data set covered a wide range of tree species composition, crown closure, height and age classes (Table 1). DBH ranged from 12.6 to 217.0 cm with standard deviation of 17.91 cm, while tree height ranged from 5.8 to 45.0 m with standard deviation of 7.23 m.

Table 1. Number of polygons by selected independent variables for the Arrow TSA audit data set.

Species	SPCS1	SPCS2	SPCS3	SPCS4	CC	N	AGE_CL	N
AT	1	3	0	1	20	2	70	8
B	9	3	1	0	30	5	90	18
BL	1	0	0	0	35	1	110	7
CW	0	5	5	4	40	6	130	5
E	0	0	0	2	50	11	190	9
FD	18	10	4	3	60	16	250+	3
HHW	4	2	4	2	70	8		
LW	9	9	7	0	80	1		
PL	8	5	5	1				
PY	0	1	0	0	HT_CL	N		
PW	0	0	3	1	15	8		
SE	0	12	2	2	25	28		
Total	50	50	31	16	35	14		

Note: $SPCS_x$ refers species composition extracted from the FIP file, CC refers crown closure in percent, HT_CL refers to mid-point of height class in m, AGE_CL refers to midpoint of age class in years, and N is the number of polygons.

3.2. Parameter Prediction

The Weibull parameters based on the PDF are presented and used for further analysis. The Weibull parameters varied by species and polygon; parameters a, b, and c ranged from 0.0 to 56.80, 3.23 to 56.80, and 0.69 to 59.95, respectively. The root mean square error (RMSE) ranged from 0.00 to 0.77. Among the eight tree species, *PY* had the widest variability (standard deviation of 19.43) followed by *MH* (standard deviation of 15.01); *CW* and *MH* showed a wide range of RMSE (0.0 to 0.36 and 0.0 to 0.77) (Table 2).

Table 2. Summary of Weibull parameter estimates, number of polygons (n) by tree species in Arrow TSA audit data set. [MIN refers to the minimum value, MAX refers to the maximum, STD refers to the standard deviation].

	a					b				c				RMSE			
Species	n	MIN	Mean	MAX	STD	MIN	Mean	MAX	STD	MIN	Mean	MAX	STD	MIN	Mean	MAX	STD
B	20	10.04	16.93	26.10	4.92	3.23	11.82	23.60	4.87	0.70	1.37	4.41	1.01	0.00	0.03	0.18	0.05
CW	22	4.00	19.57	49.60	9.58	5.10	12.12	42.77	8.23	0.93	1.45	6.06	1.11	0.00	0.04	0.36	0.08
FD	27	12.39	22.87	47.29	9.61	5.78	16.82	43.50	10.41	0.84	1.82	6.42	1.39	0.00	0.05	0.24	0.07
HW	18	6.42	19.11	33.50	7.11	5.00	13.30	30.10	7.18	0.88	1.60	5.02	1.10	0.00	0.07	0.32	0.08
MH	25	0.80	19.15	27.30	5.73	3.36	12.96	29.60	7.29	0.88	5.22	59.95	11.84	0.00	0.25	0.77	0.21
PL	18	0.43	16.72	35.00	7.39	5.09	12.77	27.20	7.23	0.69	2.20	9.07	2.18	0.00	0.07	0.33	0.10
PY	5	20.00	33.84	56.80	15.01	5.60	26.67	56.80	19.43	0.97	2.47	6.47	2.33	0.05	0.12	0.24	0.07
SE	16	1.40	23.44	40.00	10.59	3.95	15.65	37.50	9.53	0.91	1.82	5.32	1.33	0.00	0.06	0.25	0.08

Among the 154 species and polygon combinations (with more than four tallied trees), parameters were estimated for 151 polygons (Appendix I). This high proportion of convergence can be ascribed to the use of starting values obtained from the percentile based parameter estimation equations. The remaining species and polygon combinations failed to converge. The RMSE decreased with an increase of the number of tallied trees in each polygon (Appendix I).

Among several equations tested to estimate the number of trees per hectare (ESPH) in a polygon, the following equation was selected (*ERMSE* of 261 trees; I^2 of 0.46, n= 50 polygons):

$$ESPH = 848.496 * AGE^{-0.31184} * CC^{0.2967}$$

Where: CC refers crown closure and AGE refers to stand age. Stand age is the age of a stand at the time of classification and is extracted from the FIP file.

To illustrate the use of the parameter estimation method, Polygon 282-23 was selected. This polygon has a species composition of $PL_{80}B_{10}FD_{10}$ with crown closure of 30% and stand age of 50. The number of the stems per was estimated, as 687 using ESPH equation:

$$ESPH = 848.496 * 50^{-0.31184} * 30^{0.2967}$$

Following this, the 687 stems per hectare were prorated based on the aerial species composition (i.e., 550 stems were PL; 68 were B, 68 stems were FD. The Weibull parameters for each species were estimated using the following equations:

For PL:

$$a = 23.149 - 20.844 * persp$$

$$b = 15.267 - 8.096 * persp$$

$$c = 3.794 - 5.169 * persp$$

where: *a*, *b*, and *c* are estimated Weibull parameters, *persp* is aerial species composition as a ratio, *VOLT* is volume/ha (VDYP projected), *AGE* is stand age, and *HGTC* is the mid-point of the height class.

For B:

$$a = 21.9673 - 12.4676 * persp$$

$$b = 8.699 + 7.725 * persp$$

$$c = 2.065 - 1.7088 * persp$$

For FD:

$$a = 26.196 + 0.014 * VOLT - 30.855 * persp$$

$$b = 5.9652 + 0.12922 * AGE - 5.3317 * persp$$

$$c = 0.7912 + 0.076 * HGTC - 3.0553 * persp$$

where: *VOLT* is volume/ha (VDYP projected), *AGE* is stand age, and *HGTC* is the mid-point of the height class.

Appendix II lists the parameter prediction equations for the remaining tree species. These equations are not tested thoroughly and should be used with caution.

Using the estimated Weibull parameters, the proportion of trees in each diameter class was estimated. For example, the proportion of trees (fi_2) between diameter 22.5 and 32.5 cm was estimated as:

$$fi_2 = (\mathrm{E}XP(\text{-}((32.5\text{-}\,\mathrm{a})/\mathrm{b})^{\mathrm{c}})\text{-}\,\mathrm{EXP}(\text{-}((22.5\text{-}\,\mathrm{a})/\mathrm{b})^{\mathrm{c}})$$

The frequency was then calculated by multiplying the estimated stems per hectare for the species by the respective proportion (Figure 1).

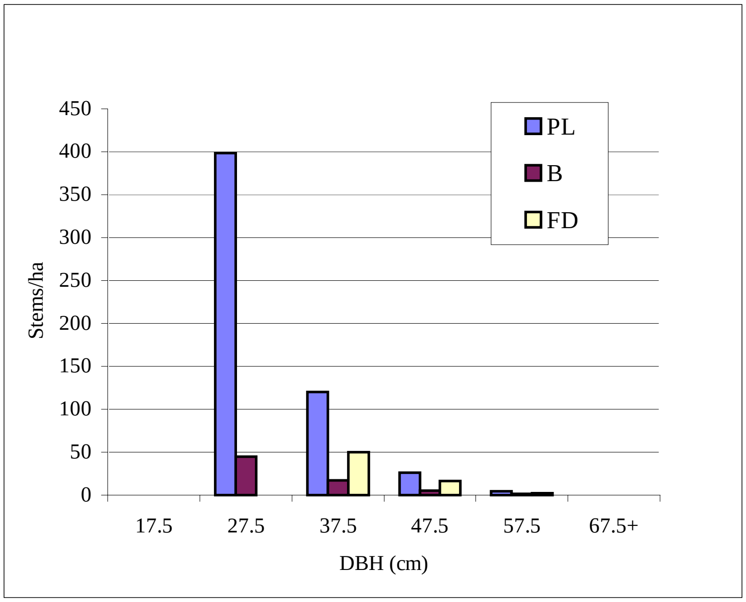

Figure 1. Estimated frequency by DBH using parametric prediction method for Polygon 282-23.

3.3. Most Similar Neighbor Method (MSN)

The MSN analysis used 13 ground and 23 aerial variables for each polygon. The ground variables for the reference polygons ranged from 282 to 1352 stems per hectare, 10.6 to 67.0 m^2/ha in basal area, and 13.0 to 217.0 cm in DBH (Table 3), whereas the aerial variables ranged from 14 to 34 m in site index, 20 to 80% in crown closure, and 17.0 to 43.0 cm in quadratic mean diameter. Aerial variables for the target polygons ranged from 16 to 32 m in site index, 20 to 70% in crown closure, and 16.0 to 43.0 cm in quadratic mean diameter (Table 3). These summary statistics showed that the reference and target polygons were balanced and covered similar range of tree sizes, stand attributes, and dispersions.

Table 3. Summary statistics for the ground and aerial variables for the Arrow TSA audit data.

Ground Variable	Reference polygons		n = 40					
	Min	Mean	Max	Std Dev				
BA_CW	0.0	8.1	27.0	7.4				
BA_PL	0.0	4.7	28.0	6.6				
BA_FD	1.0	10.5	37.0	8.9				
BA_MH	0.0	2.9	22.0	4.2				
MINDBH	13.0	14.9	32.0	3.1				
MAXDBH	35.0	74.0	217.0	32.0				
ONE16	14.0	16.2	35.0	3.5				
ONE63	16.0	25.9	53.0	6.9				
ONE97	30.0	48.6	85.0	12.5				
SPH	201	719	1352	282				
BA	19.0	37.3	67.0	10.6				
Aerial Variable	Reference polygons		n=40		Target polygons		n =10	
	Min	Mean	Max	Std Dev	Min	Mean	Max	Std Dev
PCT1	30.0	59.5	90.0	16.2	40.0	62.1	90.0	18.1
PCT2	10.0	27.4	40.0	10.7	10.0	26.9	40.0	12.5
HTG_CL	15.0	25.8	35.0	6.2	15.0	28.0	35.0	8.2
AGE_CL	50.0	7.2	300.0	64.5	50.0	111.7	300.0	78.2
SITE_IDX	14.0	24.2	34.0	4.7	16.0	24.4	32.0	5.3
HT_PRO	15.0	24.9	35.0	5.3	14.0	25.4	31.0	6.1
CRWN_CL	20.0	52.6	80.0	13.7	20.0	51.0	70.0	17.3
VOLT	100.0	472.7	938.0	193.3	173.0	490.1	890.0	218.6
QMD	17.0	26.3	43.0	5.5	16.0	26.5	43.0	7.5

Table 4. Attributes of target versus selected reference polygons (**in bold**) for the Arrow TSA audit data set. SPSCx refers to species composition, CC is crown closure, PCTx refers to percent composition of species x, QMD is quadratic mean diameter in cm (VDYP projected), BEC refers to the biogeoclimatic zone, HGTC is midpoint of height class, AGEC is midpoint of age class, SI is site index in m, and VOLT is volume/ha (VDYP projected). SPH and BAHA are number of trees and basal area per ha from ground measurements.

Polygon	HGTC	AGEC	SI	SPCS1	SPCS2	SPCS3	SPCS4	PCT1	PCT2	PCT3	PCT4	PCT5	CC	BEC	QMD	VOLT	SPH	BAHA
73-47	25	70	27.6	FD	L	PW	PL	50	30	10	10	0	60	ICH	22.3	473.6	587	28.0
355-32	**25**	**70**	**25.0**	**L**	**FD**	**PL**	**E**	**50**	**30**	**10**	**10**	**0**	**70**	**ICH**	**22.9**	**379.0**	**863**	**26.2**
119-21	35	195	23.1	B	S			60	40	0	0	0	60	ESSF	31.1	560.7	854	56.7
288-28	**35**	**195**	**25.3**	**B**	**S**			**64**	**36**	**0**	**0**	**0**	**35**	**ICH**	**31.6**	**617.9**	**525**	**49.3**
144-19	35	110	28.8	FD	L			90	10	0	0	0	60	ICH	30.9	515.6	700	50.0
345-14	**25**	**90**	**29.7**	**FD**	**L**			**90**	**10**	**0**	**0**	**0**	**70**	**ICH**	**26.8**	**605.0**	**1232**	**42.7**
173-16	15	110	15.8	FD	PL	L		40	40	20	0	0	20	ICH	20.2	173.1	290	29.3
604-48	**35**	**195**	**23.6**	**H**	**FD**	**S**	**PW**	**40**	**30**	**10**	**10**	**10**	**30**	**ICH**	**39.1**	**229.9**	**608**	**30.0**
191-45	35	110	28.3	FD	H	PW	CW	50	30	10	10	0	70	ICH	26.5	687.7	1221	50.7
215-42	**25**	**90**	**23.7**	**FD**	**H**	**CW**	**S**	**40**	**30**	**20**	**10**	**0**	**60**	**ICH**	**26.5**	**404.7**	**1084**	**56.0**
194-18	25	110	24.3	FD	L			60	40	0	0	0	60	ICH	24.7	409.6	298	18.2
185-43	**25**	**90**	**24.6**	**FD**	**AT**	**CW**		**50**	**30**	**20**	**0**	**0**	**60**	**ICH**	**23.8**	**371.3**	**817**	**38.3**
435-15	35	250+	22.9	H	S	B	CW	50	30	10	10	0	60	ICH	42.8	890.1	283	41.1
274-36	**35**	**250+**	**23.0**	**HW**	**CW**	**SE**	**FD**	**50**	**30**	**10**	**10**	**0**	**60**	**ICH**	**42.9**	**871.9**	**201**	**28.6**
282-23	15	70	15.8	PL	B	FD		80	10	10	0	0	30	ESSF	16.3	185.6	981	34.2
446-12	**25**	**90**	**24.5**	**L**	**PL**	**FD**	**H**	**40**	**20**	**20**	**10**	**10**	**80**	**ICH**	**23.0**	**377.5**	**701**	**42.0**
304-50	25	90	25.0	L	FD			90	10	0	0	0	30	ICH	23.3	388.8	505	32.0
89-41	**25**	**130**	**24.3**	**L**	**FD**	**PL**		**60**	**30**	**10**	**0**	**0**	**50**	**ICH**	**26.7**	**412.1**	**725**	**52.4**
724-17	35	90	32.0	LW	FD	CW	SE	51	29	8	6	6	60	ICH	26.7	614.3	959	54.2
121-7	**25**	**70**	**24.2**	**L**	**FD**	**PL**	**E**	**50**	**30**	**10**	**10**	**0**	**50**	**ICH**	**20.5**	**289.1**	**745**	**30.9**

The relationships between ground and aerial variables were considered in selecting a reference stand. The canonical correlation report showed that nine canonical vectors explained 90% of the variance. Age-class midpoint, site index, projected heights, and volume per hectare were given higher weights in obtaining nearest neighbors.

There was good correspondence in aerial variables between the target polygon and the selected nearest neighbor (Table 4). There were two mismatches of BEC zone (i.e., Polygons 288-28 and 446-12 (ICH) were selected as a reference stand for Polygons, 119-21 and 282-23 (ESSF)).

In the MSN software, the validation statistics were computed by comparing the observed (the reference polygon) with the predicted values from the second-most similar neighbor (the most similar neighbor being itself). The validation statistics table reports the mean, standard deviation the observed and predicted ground, and also aerial variables, and their differences (residuals, Table 5). There was no significant difference between the observed and predicted values for any of the variables at the $\alpha = 0.05$ probability level (Table 5). This indicates that the selected reference polygons are reasonable estimate for the target stands (Moeur 1995).

Table 5. Validation results for the reference and target polygons for the 40 polygons used to derive the MSN. [*p* value refers the probability of the predictions differing from the observed values obtained from a paired t-test].

Inventory Variable	Observed		Predicted		Residual		Paired t-test
	MEAN	STD	MEAN	STD	MEAN	STD	p value
BA_CW	8.10	7.42	7.10	6.70	0.04	0.32	0.52
BA_PL	4.73	6.59	5.13	6.82	0.01	0.20	0.70
BA_FD	10.48	8.87	12.10	10.99	0.05	0.27	0.28
BA_MH	2.93	4.16	2.90	4.88	0.00	0.22	0.98
MINDBH	14.93	3.15	15.48	4.21	0.03	0.22	0.32
MAXDBH	73.98	32.05	69.73	28.30	0.02	0.22	0.45
SPH	719	282	748	233	0.03	0.20	0.54
BA	37.30	10.62	38.03	11.50	0.02	0.27	0.81
PCT1	59.48	16.20	61.60	15.94	0.04	0.33	0.69
PCT2	27.40	10.70	26.65	10.85	0.03	0.40	0.81
CC	52.63	13.73	53.88	13.70	0.02	0.26	0.69
VOLT	472.68	193.31	454.38	186.23	0.02	0.16	0.72
QMD	26.25	5.51	26.03	5.05	0.01	0.17	0.90

3.4. Comparison of Approaches

The predictive ability of the MSN and parameter prediction methods was evaluated by comparing the similarities and differences of the observed and estimated species composition (Table 6).

Table 6. Summary of observed (ground) and estimated number of trees per hectare by species and method for the 10 randomly selected target polygons. SPH is number of trees/ha for the target polygon, *ESPH* is estimated number of trees/ha, and MSPH is number of trees/ha for the reference polygon.

Target polygon	Observed frequencies		Parameter Prediction		Reference Polygon selected using MSN	
	SPH	SPH by species	ESPH	ESPH by species	MSPH	SPH by species
73-47	587	FD PL MH 365 142 28	844	FD MH PL PW 422 253 84 84	863	PL FD MH HW CW PW 248 179 143 140 74 11
119-21	854	B SE HW PL CW 480 320 29 19 5	548	B SE 329 219	525	B SE CW FD PL HW 330 93 33 32 28 10
144-19	700	FD PL MH PW 513 165 11 11	703	FD MH 632 70	1232	FD CW HW MH PW SE 540 257 224 85 82 45
173-16	290	FD B PL SE MH 134 47 3 3 2	507	FD PL MH 203 101 203	608	CW HW FD MH PW SE 262 150 81 52 41 19
191-45	1221	HW FD PW CW MH SE PL 676 211 128 86 51 41 29	736	FD HW PW CW 368 221 74 74	1084	HW FD CW B SE 411 332 300 27 14
194-18	298	FD PL PY MH CW 155 53 47 40 4	703	FD MH 422 281	817	CW MH B HW 445 196 120 57
435-15	283	SE HW FD B CW 138 62 49 23 11	483	HW SE B CW 241 145 48 48	201	SE B HW CW 99 59 43 1
282-23	981	B PL FD PW SE 496 425 32 23 4	687	PL B FD 550 69 69	701	PL MH B SE FD CW PW 180 168 134 120 53 25 21
304-50	505	B CW SE FD PL MH 90 68 44 19 15 10	619	MH FD 557 62	725	CW HW SE PL FD PW B MH 222 158 61 38 37 22 77 5
724-17	959	CW B HW MH FD SE PW 372 252 147 96 63 21 7	710	MH FD CW SE 356 213 71 71	745	CW FD SE MH 274 244 43 21

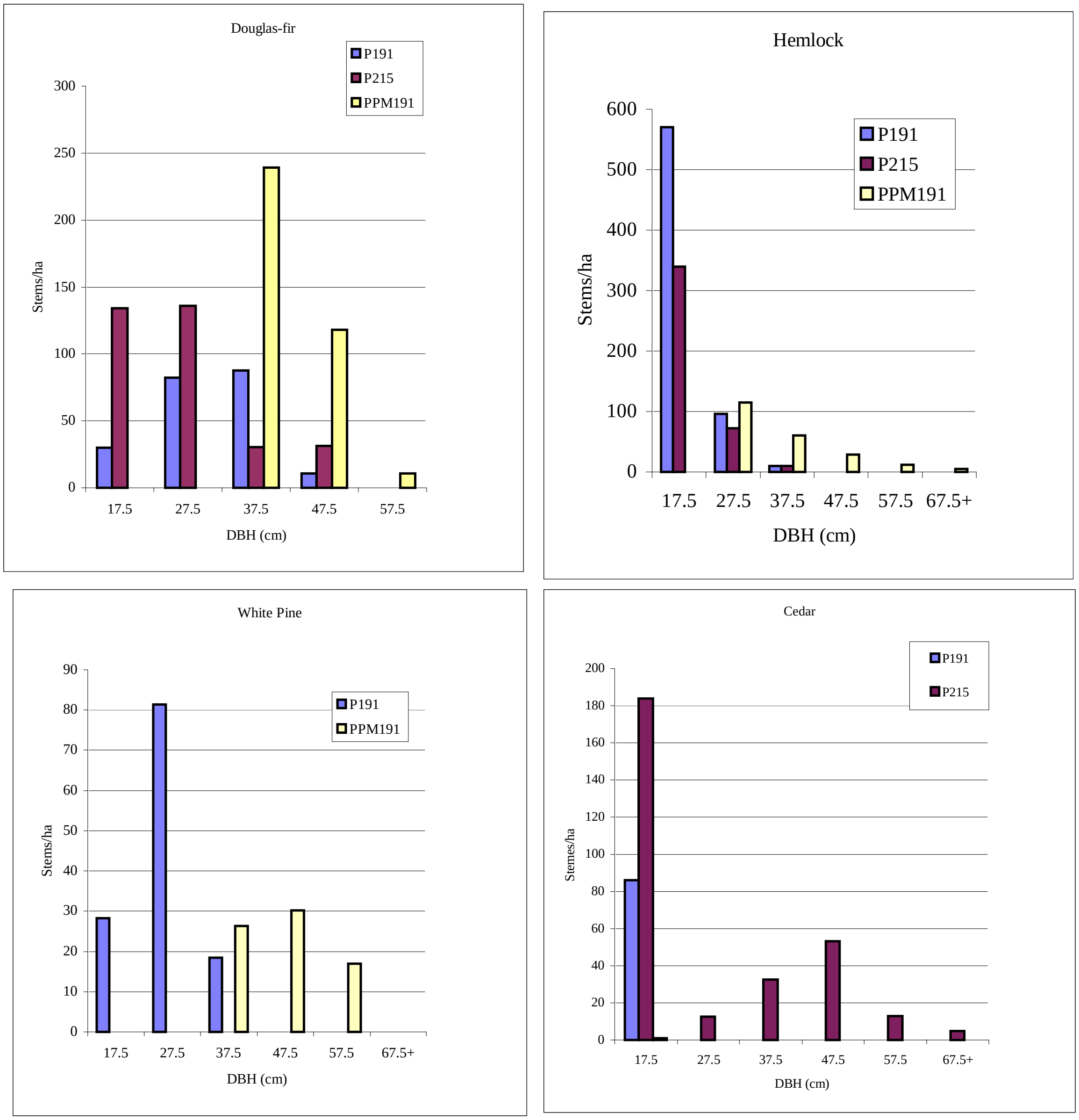

Figure 2. Observed and estimated diameter frequencies by species for Polygon 191-45. Blue bars are based on ground data, yellow bars are based on the parameter prediction method, and pink bars are based on the ground data from reference polygon 215-42 selected by the most similar neighbor method.

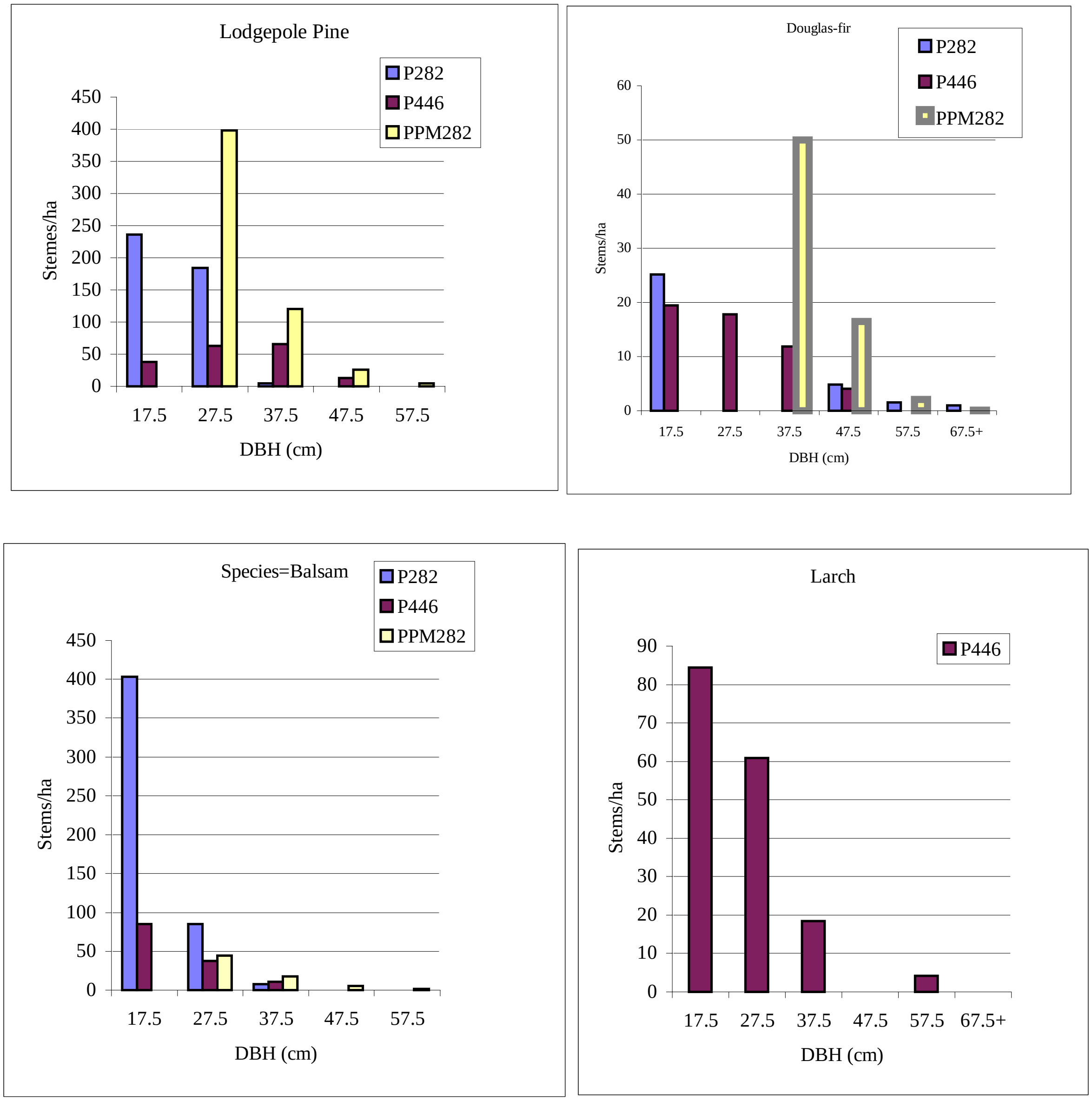

Figure 3. Observed and estimated diameter frequencies by species for Polygon 282-23. Blue bars are based on ground data, yellow bars are based on the parameter prediction method, and pink bars are based on the ground data from reference Polygon 446-12 selected by the most similar neighbor method.

4. CONCLUSIONS AND RECOMMENDATIONS

Parameter prediction and the Most Similar Neighbor (MSN) tree list generation approaches were illustrated using an example of an existing audit data set. Some of the issues that arose include:

1. The cluster of sample plots in each polygon of the audit data set may not represent the polygon well. Since the aerial data for the entire polygon are used for the MSN approach, the selected reference polygon tree list may not well represent the tree list for the entire target polygon. For the parameter prediction approach, the estimated parameters may not represent the entire polygon.

2. The Weibull parameters were not stable for some of the species and polygon combinations. This can be ascribed to the limited number of tallied trees for a particular species and polygon combination.

3. Due to time constraints, more effort on various starting values for the Weibull PDF was not made. Weibull parameters may therefore not be the minimum least squares values.

Based on this one example data set, the literature review, and the TLGW, recommendations are:

1. *We recommend a combined use of the MSN and the parameter prediction methods for tree list generation, and linking GY models to aerial information*.

The combined MSN and parameter prediction approach consider the size of available data and provide sound and efficient projections, as suggested at the TLGW workshop.

2. *We suggest appropriate documentation of the tree list generation procedure from start to finish so that other forestry practitioners can use the MSN and the parameter prediction methods in their data sets.*

A well-documented tree list generation procedure encourage other practitioners to generate tree list, and to link GY models to aerial information.

3. *We recommend that larger data set be used for generating tree lists and linking GY models to aerial information*.

Combining audit polygons for two or more TSAs will provide a wide range of forest types and conditions. This may also allow preliminary aggregation of polygons prior to analyses, in order to prevent the problem that results in having one cluster of plots representing the polygon.

4. *We recommend further testing of both the parameter prediction and the MSN method. Testing should include altering the ground (X set) and aerial (Y set) variables for the MSN analysis and altering fitting techniques and model forms for the parameter prediction method.*

The results of the MSN analysis obtained in this study could totally different if different sets of target and reference stands were selected or if different sets of ground (X set) and aerial (Y set) variables were used. Three specific recommendations for testing include:

4.1. We recognize that relationships between the Weibull parameter estimates, and the aerial attributes, including the proportion of species, can be obtained by using various fitting methods. We recommend that weighted least squares (WLS) and seemingly unrelated regression (SUR) fitting techniques, and various transformations of the aerial stand attributes be assessed. Probability density functions such as S-B Johnson's distribution be considered.

The Weibull parameters were poorly related to aerial stand attributes. This poor relationship could be attributed to the bi-modal or multi-modal diameter distribution of some of the tree species. The Weibull PDF does not characterize these types of diameter distributions. The proportion of species (% species composition) was not significantly (linearly) related to the Weibull parameter estimates. This implies that the shape of the Weibull curves were similar regardless of species composition in a subject polygon. However, this indication is not logical.

4.2. We recommend that an accurate stems per hectare prediction model be developed.

The RMSE for the selected stems per hectare model was 261 trees per hectare and this could be improved by fitting other model forms and combination of variables.

4.3. We suggest additional studies on the use of K-nearest Neighbor (k-nn) method and weighted averages of tree lists. This might provide better match of species composition and diameter frequency between the observed ground data and forest map labels.

The K-nearest Neighbor (k-nn) and parameter recovery approaches were not included in this report due to time constraints.

5. *Generating tree lists by biogeoclimatic (BEC) zone rather than by timber supply area (TSA) and by species group or species guild rather than by individual tree species might result in better estimates.*

Combining audit data by BEC zone and by species group will provide higher number of tallied trees for each species and BEC zone combination.

6. *Additional ground variables and aerial information (e.g., landsat imagery) should be obtained and the advantages of using these additional variables should be made.*

Ground variables such as maximum basal and crown competition factors (used by Prognosis^{BC}) may assist in generating tree lists and in linking GY models to aerial information. Other remotely sensed data, in addition to aerial variables from forest cover data, might provide a better similarity function in selecting the most similar neighbor polygon in the MSN analysis and also some of the variables might be useful in predicting the Weibull parameter estimates.

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APPENDIX I

Weibull parameter estimates, number of tallied trees by tree species and polygon in Arrow TSA.

Polygon	SAMPLE	Species	a	b	c	No. of tallied trees	RMSE
7	6	B	13.74	12.28	0.92	32	0.00
7	6	SE	1.40	32.69	2.12	41	0.01
26	10	MH	19.80	17.06	4.28	12	0.32
40	27	FD	20.00	5.81	1.00	7	0.05
40	27	PL	8.70	9.03	0.69	37	0.00
53	1	B	10.55	11.55	0.72	56	0.00
53	1	SE	27.80	12.18	0.99	14	0.03
58	40	MH	15.00	8.90	3.59	11	0.04
58	40	PL	14.80	5.09	1.00	8	0.05
73	47	FD	15.00	15.10	1.94	30	0.02
73	47	MH	17.30	14.50	3.66	5	0.77
73	47	PL	19.87	6.03	1.03	7	0.03
76	4	FD	13.90	8.27	1.00	24	0.01
76	4	MH	22.50	14.85	13.32	8	0.32
76	4	PY	56.80	56.80	6.47	9	0.14
82	38	B	13.35	13.77	0.84	41	0.00
89	41	CW	18.50	9.50	0.99	17	0.02
89	41	FD	33.60	32.60	3.22	6	0.24
89	41	HW	17.00	5.38	0.99	5	0.09
89	41	MH	27.30	29.60	2.27	16	0.32
89	41	PL	35.00	26.10	5.60	5	0.32
89	41	SE	23.30	22.90	1.91	6	0.25
95	24	CW	13.80	6.82	1.00	13	0.01
95	24	HW	18.40	13.30	1.77	11	0.06
95	24	MH	15.00	3.97	1.01	6	0.58
119	21	B	14.60	10.30	0.99	20	0.01
121	26	B	20.00	15.18	0.99	28	0.00
121	26	SE	35.00	10.23	1.00	13	0.03
121	7	CW	15.00	7.60	1.69	11	0.04
121	7	FD	16.97	5.78	1.03	13	0.02
121	7	MH	16.50	12.60	1.81	11	0.41
144	19	FD	15.00	13.11	0.99	37	0.01
144	19	PL	17.78	5.19	2.14	6	0.06
165	34	CW	14.90	6.43	1.00	9	0.01
165	34	MH	17.90	10.20	1.37	7	0.56
165	34	PL	18.80	6.40	1.00	10	0.05
173	16	B	26.10	5.87	1.00	8	0.06
173	16	FD	15.70	27.60	1.50	32	0.02
173	16	MH	20.00	11.45	4.87	22	0.30
185	43	B	18.70	12.10	3.90	6	0.18
185	43	CW	14.96	11.99	0.97	28	0.00
185	43	HW	25.70	17.60	5.02	5	0.32
185	43	MH	20.00	14.87	2.03	12	0.32
186	37	CW	14.60	8.12	1.00	16	0.01
186	37	FD	17.40	13.10	0.99	25	0.01
186	37	MH	15.70	4.61	0.99	5	0.10
191	45	FD	19.40	20.90	3.02	19	0.03
191	45	HW	13.01	11.92	0.88	22	0.00
191	45	PY	22.49	5.60	1.01	8	0.05

194	18	FD	18.50	10.45	1.00	17	0.01
194	18	MH	24.70	8.69	2.32	6	0.08
194	18	PL	20.00	17.60	2.54	7	0.11
194	18	PY	20.00	27.60	1.37	10	0.08
215	42	CW	15.00	7.41	0.99	25	0.02
215	42	FD	20.00	12.51	1.00	23	0.00
215	42	HW	14.80	7.91	1.00	12	0.01
253	31	CW	13.30	5.10	2.26	7	0.06
253	31	FD	17.90	5.94	1.00	9	0.03
253	31	MH	19.30	3.36	1.00	5	0.14
253	31	PL	9.84	9.37	0.82	20	0.00
272	30	B	14.49	12.72	0.93	27	0.00
272	30	SE	13.85	17.48	2.43	20	0.03
274	36	B	25.85	11.59	2.19	6	0.06
274	36	HW	26.50	24.90	1.86	9	0.14
274	36	SE	29.77	18.96	0.91	34	0.01
278	20	B	16.20	12.46	0.99	29	0.00
278	20	SE	4.71	24.15	5.32	16	0.04
282	23	B	10.23	10.71	0.76	34	0.00
282	23	FD	47.29	12.25	1.01	5	0.02
282	23	PL	11.54	11.74	0.79	36	0.00
288	28	B	14.80	9.13	1.00	21	0.01
288	28	CW	49.60	17.12	0.98	26	0.03
288	28	FD	35.00	27.60	6.42	6	0.20
288	28	SE	31.59	10.80	1.01	18	0.02
294	22	B	16.38	3.23	1.05	5	0.09
294	22	PL	9.43	9.95	0.71	42	0.00
304	50	B	25.00	22.60	4.41	12	0.07
304	50	MH	17.40	12.13	11.97	22	0.16
304	50	SE	23.59	3.95	3.14	5	0.13
336	35	CW	20.00	18.60	1.60	32	0.01
336	35	FD	40.00	16.42	1.97	11	0.09
336	35	HW	6.42	23.90	2.63	25	0.02
336	35	MH	25.00	18.70	0.97	10	0.28
336	35	PY	40.00	14.35	0.97	5	0.10
345	14	CW	16.99	8.50	1.01	12	0.01
345	14	FD	13.02	13.28	0.84	35	0.00
345	14	HW	15.00	7.60	1.66	9	0.06
351	9	B	15.40	23.60	1.38	30	0.02
351	9	CW	18.80	12.60	1.88	8	0.07
355	32	CW	14.70	7.50	1.57	5	0.15
355	32	FD	15.00	8.70	0.99	16	0.01
355	32	HW	14.49	5.00	1.01	8	0.03
355	32	MH	13.80	5.94	0.99	14	0.10
355	32	PL	13.60	8.39	1.00	14	0.00
355	44	CW	29.50	23.70	2.00	5	0.36
355	44	FD	20.00	12.91	1.00	18	0.00
355	44	MH	0.80	21.81	59.95	14	0.10
355	44	PL	16.50	14.60	1.95	18	0.03
382	11	CW	29.86	10.15	0.93	11	0.03
382	11	FD	20.00	17.38	0.98	23	0.02
404	13	CW	37.49	10.62	1.01	8	0.02

404	13	MH	25.20	19.20	4.54	8	0.65
404	13	PL	20.00	12.60	2.08	18	0.02
404	13	SE	17.83	12.85	0.98	18	0.00
414	25	B	10.04	11.04	0.70	49	0.00
414	25	PY	29.90	29.00	2.51	6	0.24
414	25	SE	18.70	9.60	0.98	11	0.04
431	8	B	15.20	9.60	1.60	12	0.04
431	8	CW	19.90	10.65	0.98	11	0.04
431	8	HW	13.20	8.40	1.00	15	0.01
431	8	MH	13.32	12.36	0.88	22	0.00
435	15	FD	40.00	43.50	2.54	11	0.12
435	15	HW	31.60	9.17	0.98	7	0.09
435	15	SE	27.90	9.02	1.00	13	0.03
440	39	MH	26.30	28.70	2.28	9	0.14
440	39	PL	20.00	27.20	1.58	9	0.09
440	39	SE	19.98	10.92	0.93	19	0.02
446	12	B	21.00	4.70	1.00	5	0.08
446	12	FD	28.90	9.42	5.16	6	0.11
446	12	MH	19.97	4.27	1.06	9	0.04
446	12	PL	20.52	13.30	2.02	14	0.03
446	12	SE	24.59	4.01	1.34	5	0.04
507	3	FD	19.42	16.25	0.94	30	0.00
507	3	HW	20.00	17.60	1.76	8	0.15
507	3	MH	26.40	11.03	0.98	5	0.09
507	3	PL	0.43	26.95	4.60	16	0.03
578	33	CW	19.81	16.38	0.96	19	0.00
578	33	FD	12.39	8.00	1.70	15	0.02
578	33	MH	17.50	8.08	0.98	5	0.05
578	33	PL	23.30	11.90	9.07	5	0.33
579	29	B	15.40	13.10	1.08	19	0.03
579	29	HW	20.00	16.48	0.96	8	0.05
579	29	SE	40.00	37.50	4.17	6	0.24
585	46	CW	4.00	42.77	6.06	20	0.04
585	46	FD	35.00	42.60	2.98	12	0.10
585	46	HW	33.50	30.10	3.37	7	0.19
585	46	SE	35.00	13.13	0.98	15	0.04
604	48	CW	14.88	7.40	0.98	13	0.01
604	48	FD	18.50	17.72	0.95	6	0.06
604	48	HW	26.10	12.35	1.00	19	0.01
605	2	CW	19.99	9.79	1.01	19	0.01
605	2	MH	17.10	5.35	0.98	33	0.18
642	5	FD	19.50	9.35	0.98	11	0.04
642	5	HW	15.39	6.49	1.01	11	0.02
642	5	PL	20.90	8.43	0.99	11	0.04
724	17	B	21.50	10.90	1.00	13	0.01
724	17	CW	14.89	7.90	0.98	22	0.01
724	17	FD	30.00	27.60	2.96	7	0.19
724	17	HW	20.00	9.46	0.98	8	0.05
724	17	MH	25.00	21.70	2.39	9	0.13
2009	49	HW	12.85	11.90	0.88	25	0.00

APPENDIX II

Weibull parameter prediction equations by species.

For CW:

$$a = 39.696 - 11.33 * persp - 0.4999 * CC + 0.01915 * VOLT$$

$$b = 2.406 + 1.17 * persp - 0.019 * VOLT$$

$$c = 1.608 - 0.584 * persp$$

where: *a*, *b*, and *c* are estimated Weibull parameters, *persp* is aerial species composition as a ratio, *VOLT* is volume/ha (VDYP projected), and CC is crown closure.

For HW:

$$a = 15.54 - 17.01 * persp + 0.06 * AGE$$

$$b = 3.195 + 0.018 * VOLT + 1.22 * persp$$

$$c = 2.21 - 2.65 * persp$$

where *AGE* is stand age.

For MH:

$$a = 19.03 + 0.912 * persp$$

$$b = 12.94 + 0.13 * persp$$

$$c = 40.50 - 1.98 * HGTC + 0.038 * VOLT - 16.35 * persp$$

where *HGTC* is the mid-point of the height class.

For PW and PY:

$$a = 34.05 - 1.39 * persp$$

$$b = 7.19 - 129.94 * persp$$

$$c = 0.64 - 12.192 * persp$$

For SE:

$$a = 29.073 - 19.17 * persp$$

$$b = 10.996 + 15.83 * persp$$

$$c = 2.26 - 1.475 * persp$$