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Compatibility Improvements and Bias Reduction in Height-Age Models

Gordon D. Nigh

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ABSTRACT

Inconsistencies between breast height age and adjustments for sectioning bias in stem analysis data may cause a bias in height–breast height age prediction systems. A further bias may be introduced when applying the model if the breast height age of the tree is taken to be the ring count at breast height. These biases can be removed by making the model and data compatible with the definition of breast height age.

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1 INTRODUCTION

Stem analysis data are frequently used to develop height-age models that predict height from site index (SI) and age. To obtain these data, a tree is felled and crosscuts are made at various points along the stem. At each cut, the height and annual ring count (from which age is calculated) are recorded. Annual nodes (points on the stem where the annual height growth ceases) are generally avoided when making the crosscuts. On average, the crosscuts lie midway between nodes. If the nodes are assumed to lie at the crosscuts, a bias, or error, is introduced into subsequent calculations. Several methods have been proposed to remove this bias when determining the height of the tree at a given total age. Dyer and Bailey (1987) tested six of these methods and concluded that Carmean's (1972) correction was the most accurate for removing the bias.

Breast height age (BHA) is often preferred over total age because it eliminates "early erratic growth that is poorly related to site quality that occasionally occurs before a tree reaches breast height" (Monserud 1984: 947). Breast height age is determined from the number of annual rings at breast height (1.3 m). To reconstruct a tree's height growth pattern, the breast height age for each stem section is determined by subtracting the section's annual ring count from the ring count at the current breast height. However, in some cases this procedure yields inaccurate results.

This paper shows how models for predicting height from breast height age and site index can be biased if the definition of breast height age and the model's development data, formulation, and application are not compatible. A definition of breast height age and model formulation that reduces bias is recommended for future modelling.

2 BACKGROUND

2.1 STEM ANALYSIS BIAS AND FIXED INTERCEPT MODELS

Stem analysis has been the primary method of collecting data for height-age models in British Columbia. Previously, the age of a tree at breast height was calculated by subtracting the stem section ring count from the number of rings at breast height. This was the breast height age when the tree's leader surpassed

the height of the sectioning point. To correct sectioning bias (Carmean 1972), the heights at the sections were adjusted upwards so that they corresponded, on average, to the height at a node. The magnitude of this adjustment depends on the tree's growth rate, which in turn is related to site index — the amount of the adjustment usually increases as site index increases. An alternative correction method is to subtract half a year from the breast height ring count (Jim Goudie, B.C. Min. For., Res. Branch, pers. comm., 1994).

An error or bias can arise in height–breast height age models, particularly those with a fixed intercept formula as in equation (1).

$$\hat{H} = 1.3 + f(BHA, SI) \quad (1)$$

where: \hat{H} = estimated height, and

$f(BHA, SI)$ = a function that describes the height growth pattern above breast height.

Function f is usually asymptotic; that is, it approaches a constant (0) as BHA approaches 0. Therefore, \hat{H} will approach 1.3 m as BHA approaches 0. It may also be doubly asymptotic, approaching some maximum value as BHA goes to infinity.

2.2 STEM ANALYSIS BIAS AND DEFINITIONS OF BREAST HEIGHT AGE

1 The easiest way to handle measurements taken while the trees are still growing in height is to measure the height to the end of the previous growing season and to ignore the outermost annual ring. The other option is to assume that the tree is midway through the growing season and to account for the half growing season in the data. This option is not considered here.

Bias occurs, particularly at young ages, if the model, data, and definition of breast height age are not compatible. Three possible definitions of breast height age are described below, along with a description of the corresponding bias that may arise if equation (1) is fit to the stem analysis data. In the following discussion, I assume that the trees have finished their height growth for the year,¹ that an asymptotic function is used for $f(BHA, SI)$, and that stem sections are always taken midway between nodes. Figures 1–3 may be consulted to help clarify the different definitions of breast height age.

2.2.1 Definition One: Breast height age is the number of *complete* rings at breast height (Figure 1).

The tree's breast height age is its ring count at breast height minus one because the innermost ring is a partial ring. The true

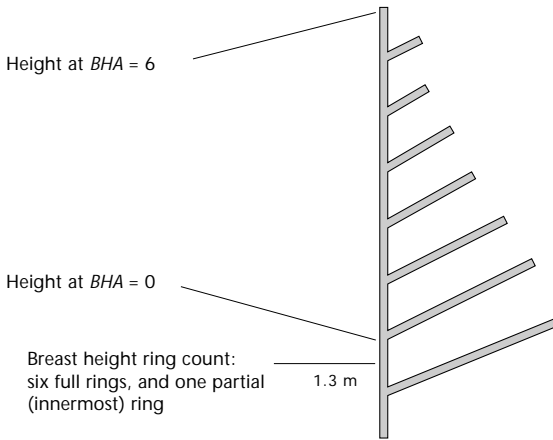


FIGURE 1 *Breast height age definition one: 6-year-old (BHA) tree. This definition ignores the first partial year of growth above breast height.*

height of the tree (H) at breast height age zero is the height of the first node above breast height, making model (1) biased at $BHA = 0$ ($H > \hat{H} = 1.3$ m at $BHA = 0$).

2.2.2 **Definition Two:** Breast height age is the number of years the tree has been growing since it reached breast height (Figure 2).

Breast height age is calculated by subtracting 0.5 from the breast height ring count because, on average, the innermost ring only represents 0.5 years of growth. Model (1) is not biased ($H = \hat{H} = 1.3$ m at $BHA = 0$), but the data are not consistent with the definition. The inconsistency arises because the bias correction adjusts the height at each section point upwards, resulting in less desirable estimates of the model parameters.

2.2.3 **Definition Three:** Breast height age is the number of *complete* and *incomplete* rings at breast height (Figure 3).

Breast height age is determined by simply counting the rings at breast height. Model (1) is again biased because the tree is 1.3 m

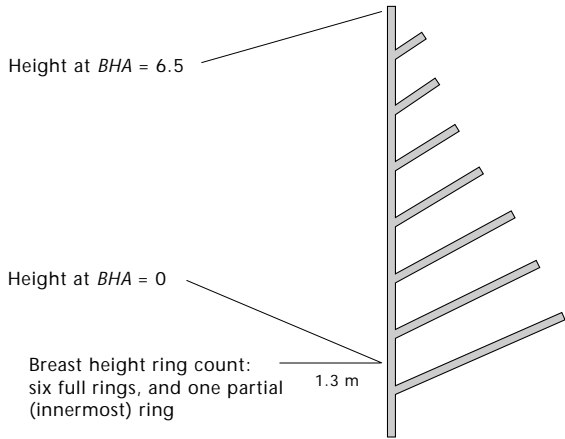


FIGURE 2 *Breast height age definition two: 6.5-year-old (BHA) tree. This definition explicitly recognizes that the innermost ring represents only 0.5 years of growth.*

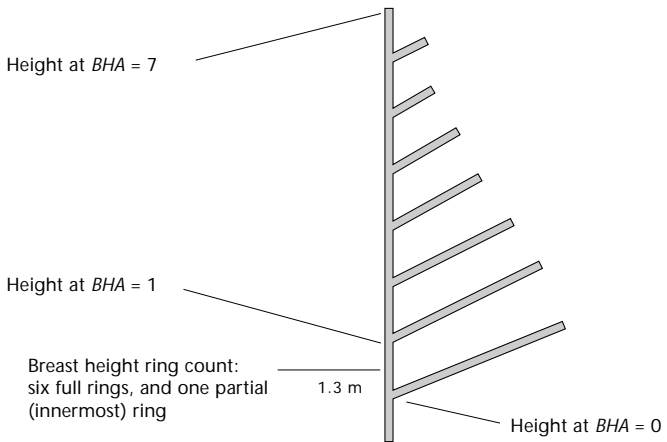


FIGURE 3 *Breast height age definition three: 7-year-old (BHA) tree. This definition considers the innermost ring to represent a full year, but the tree has only been growing 0.5 years after reaching breast height.*

tall at breast height age 0.5. At breast height age zero, the tree will be shorter than 1.3 m ($H < 1.3 \text{ m} = \hat{H}$ at $BHA = 0$).

An application bias may arise with the first two definitions of BHA. To illustrate this, suppose the model, data, and definition of breast height age are consistent. An increment core is taken at breast height and the ring count is taken to be the tree's age. This ring count is incorrect because 1.0 or 0.5 should be subtracted from the count for definitions one or two, respectively.

A brief review of the literature reveals cases where authors have unwittingly introduced this bias; for example, see Carmean and Lenthall 1989, Goelz and Burk 1992, and Ker and Bowling 1991. Fortunately, this error is small and it diminishes as breast height age increases because the only fixed point in the model is at $BHA = 0$. In other cases, it is often difficult to tell exactly how the authors defined breast height age; for example, see Biging (1985), Cieszewski and Bella (1989), and Thrower and Goudie (1992).

3 ENSURING COMPATIBILITY BETWEEN MODELS AND DEFINITIONS

Any potential biases can be eliminated by making the model and data (both model development and application) consistent with the definition of breast height age. There are three possible ways to ensure overall compatibility:

1. use a floating intercept model formulation,
2. do not correct the stem analysis data for sectioning bias, or
3. adjust the ring counts in the formulation to obtain the correct age.

The appropriate method depends on the definition of breast height age.

3.1 DEFINITION ONE

Stem analysis data are corrected for sectioning bias, as described by Newberry (1991). The breast height age of each stem section is determined by subtracting a section's ring count from the breast height ring count. The breast height age for the tip of the stem is obtained by subtracting 1 year from the breast height ring count. The height of a tree at $BHA = 0$ is the height of the first node above breast height. Therefore, this height

depends on the productivity of the site; trees growing on a site with a higher site index will be taller at breast height age zero. The following model formulation (2) should be used to represent the height growth pattern of the trees:

$$\hat{H} = [1.3 + g(SI)] + f(BHA, SI) \quad (2)$$

The term $[1.3 + g(SI)]$ is the floating intercept, so $g(SI)$ should be an increasing function of SI to reflect the correlation between the amount of the adjustment and site quality.

3.2 DEFINITION TWO

A correction for sectioning bias should not be applied to the data. The breast height age of each stem section is the difference between the section's ring count and the breast height ring count. The breast height age for the tip of the stem is the breast height ring count minus 0.5 years. Equation (1) gives the correct formulation for the model.

3.3 DEFINITION THREE

A correction for sectioning bias should be applied to the data (Newberry 1991). The breast height age of a stem section is obtained by adding one to the difference between the section's ring count and the ring count at breast height. The breast height age of the stem tip is the breast height ring count. Equation (3) gives the proper formulation of the model.

$$\hat{H} = 1.3 + f(BHA - 0.5, SI) \quad (3)$$

In this formulation, 0.5 is subtracted from the breast height age since the trees are 1.3 m tall at $BHA = 0.5$. This operation is required because function f is asymptotic to 0 as BHA approaches 0.5. One disadvantage of this formulation is that predicted heights corresponding to breast height ages less than 0.5 are undefined.

4 DISCUSSION

The definition of breast height age influences all stages of model development and use from formulating the model to

cleaning and adjusting the data used to develop the model, and finally in applying the model. Husch et al. (1972: 67) define age at the breast height point to be “the number of years the tree has been growing above the point.” This corresponds to definition two above. This definition is advantageous because, when the models are developed, an adjustment for sectioning bias does not have to be made and a simple fixed intercept model can be used. However, applying the model is complicated because breast height age is calculated by subtracting 0.5 from the ring count at breast height. The definition of breast height age used by British Columbian foresters and others is the number of rings at breast height (or definition three above) (Alemdag 1988; Forest Productivity Councils of British Columbia 1993). This method is complicated because it requires adjusting the data for stem analysis bias and a more involved model formulation. However, it is the easiest method to apply because the breast height ring count corresponds to the breast height age. Since computers have made complex models easier to handle, the choice between methods is governed by their ease of application. Therefore, definition three is recommended and the models and data must then be made compatible with this definition.

The definition chosen affects site index because site index is the top height of a stand at breast height age 50. Therefore, using the three definitions, the respective site index will represent 50.5, 50, and 49.5 years of growth after the trees reach breast height. Clearly, the site index associated with definition one is larger than that associated with definition two. The site index for definition three is the smallest. As with top height, the definition used for breast height age needs to be compatible with the growth and yield models that use breast height age as a parameter.

Previously, height-age modelling did not adhere to the recommended definition of breast height age. However, this should not cause concern because the shape of the height growth curves, for practical purposes, will not change. As well, the difference in site index between the definitions of breast height age is small. For example, a stand with a site index of 40 m under breast height age definition two would have a site index of ≈ 40.25 and 39.7 m under definitions one and three, respectively, based on Bruce's (1980) model.

Bias or error can be introduced into a height-age prediction system if the definition of breast height age and the model's development data, formulation, and application are not compatible. This bias can be equivalent to 0.5 or 1 full year of height growth at age 50. I have shown here how the data, and model formulation and application, can be made compatible for three definitions of breast height age. The procedure used for model development must be carefully thought out and explained, regardless of the definition of breast height age and the method of removing bias. When these models are applied, the ring count at breast height may need to be adjusted to be compatible with the model. Also, the different definitions of breast height age will affect the site index.

The recommended definition of breast height age is the number of annual growth rings at breast height (definition three). Consequently, height-age models must be consistent with this definition. Existing models may be inaccurate, but the bias is very small and occurs only at young ages. Therefore, the consistent use of definition three is required in future modelling only.

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