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Variable Growth Intercept Models for Lodgepole Pine in the Sub-Boreal Spruce Biogeoclimatic Zone, British Columbia

Gordon D. Nigh

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Province of British Columbia
Ministry of Forests Research Program

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ABSTRACT

Growth intercept models relate early average height growth of trees (the growth intercept) to site index (top height at breast-height-age 50). Typically, the growth intercept is the average height-growth during the first five growing seasons after the tree has reached breast height. This report presents a variable-growth intercept model that is based on the average height growth above breast-height for any number of years from 1 to 30. This model is more flexible than growth intercept models that use a fixed-growth period such as five years. The variable-growth intercept model has many advantages and few disadvantages over the traditional fixed growth intercept models.

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Site index is the top height of a stand¹ at a reference age and is a measure of site productivity. Estimates of site index are obtained from height-age curves, although the curves are specifically designed to show the height development pattern exhibited by site trees of a particular site index throughout their life. These curves are unreliable for estimating site index in juvenile stands; that is, stands below approximately 20 years breast height age.² Small deviations from the mean growth pattern near age zero (where the height-age curves converge) result in large errors in estimated site index (Curtis et al. 1974).

Growth intercept models give reliable site index estimates for young stands by relating the early average height growth of trees (the growth intercept) to the site index. Typically, the growth intercept is the average five year height growth above breast height. This is obtained when the distance between the first and sixth annual whorls above breast height is averaged. A growth intercept was first proposed as an index of site productivity in 1937 (Wakeley and Marrero 1958). Since then, many models that relate the growth intercept to site index have been developed (see for example, Warrack and Fraser 1955; Alban 1972; Carmean 1975). Recently, a growth intercept model was constructed for lodgepole pine (*Pinus contorta* var. *latifolia* Dougl.) that estimates site index from the average five year height growth immediately above breast height.³

This research report describes a variation of the traditional growth intercept model. A set of growth intercept sub-models are generated to estimate site index from the average of x years of height growth above breast height, where x varies from 1 to 30. This new method is referred to as *variable growth intercept* modelling to distinguish it from the traditional *fixed growth intercept* modelling.

DATA AND METHODS

The variable growth intercept model was developed using stem analysis data for lodgepole pine. These data were collected in conjunction with the data for the fixed growth intercept model. Forty-five 0.03-ha plots were established in the Sub-Boreal Spruce (SBS) biogeoclimatic zone (Meidinger and Pojar 1991) — 15 plots were established in each of the Lakes, Prince

1 U. Soderberg and G.D. Nigh. 1994. Top height definition. A report to the Technical Advisory Committee, Forest Productivity Councils of British Columbia, B.C. Min. For. 7 p. Unpubl. rep.

2 Breast height age, in this report, is the number of years a tree has been growing since it reached breast height (1.3 m). This definition differs from the one adopted by the Forest Productivity Councils of B.C. (1993), which defines breast height age as the number of annual rings at breast height.

3 J.S. Thrower and Associates Ltd. 1994. Development and application of growth intercepts for estimating site index of lodgepole pine in the Sub-Boreal Spruce biogeoclimatic zone. Silv. Br., B.C. Min. For. 20 p. Unpubl. rep.

George, and Quesnel forest districts. The plots were located to cover the range of site indices of lodgepole pine. Three trees that represented the productivity potential of the plot were felled, sectioned, and measured. The measurements included both tree ring counts and height, at breast height and at nine other equidistant points between breast height and the top of the tree.⁴ Breast height age was determined from the ring counts. The tree height at breast height age zero was taken to be 1.3 m. This eliminated the need for Carmean's (1972) correction to remove bias. The total height at breast height ages 1 to 30 and at age 50 were then interpolated from the section data. The interpolated heights of the three sample trees in each plot were averaged for each age. These averages represented 45 independent observations for each age between 1 and 30 inclusive, and were used to develop the variable growth intercept model. Site index was calculated by averaging the interpolated heights at breast height age 50.

⁴ The lower sections were also split and the annual height growth was determined from the terminal bud scars. This allowed the five-year growth intercept to be measured accurately.

An independent data set was assembled to test the models. These data were collected in the same manner as the model development data, except that four trees in a 0.04-ha plot were measured and the sampling covered the *SBS* zone, as well as the Montane Spruce, Interior Douglas-fir, Interior Cedar-Hemlock, and Engelmann Spruce-Subalpine Fir biogeoclimatic zones (Meidinger and Pojar 1991). While originally collected to develop new height-age curves for lodgepole pine,⁵ these data are wholly adequate for model testing.

⁵ J.S. Thrower and Associates Ltd. 1993. Height-age and site-index curves for lodgepole pine in British Columbia. Res. Br., B.C. Min. For. 25 p. Unpubl. rep.

The model given by equation (1) was used to relate the growth intercept to site index.

$$SI = b_{A,1} \cdot GI_A^{b_{A,2}} + \varepsilon, \quad (1)$$

where: SI = site index (m at breast height age 50),

$$GI_A = (H_A - 1.3) \cdot 100 \div A,$$

H_A = total tree height (m) at age A ,

A = breast height age (years),

$b_{A,1}$, $b_{A,2}$ = model parameters, and

ε = random error term.

The relationship between site index and average annual height growth changes with the age of the tree. Therefore, different values for parameters $b_{A,1}$ and $b_{A,2}$ are required and these will depend on the breast height age (A) used to calculate GI_A . This results in 30 sub-models, one for each age between 1 and 30.

Nonlinear least-squares regression (procedure NLIN in SAS [SAS Institute Inc. 1988]) was used to estimate the model parameters. Originally, an intercept parameter was included in model (1) but the slow convergence of procedure NLIN indicated that the model was over-parameterized. After a preliminary analysis, parameter $b_{A,1}$ was transformed so that model (1) took the form of:

$$SI = e^{b'_{A,1}} \cdot GI_A^{b_{A,2}} + \varepsilon, \quad (2)$$

where: $b'_{A,1}$ = transformed parameter, and
 e = the base for natural logarithms.

This reparameterization reduced the bias (Box 1971) and parameter effects nonlinearity (Bates and Watts 1980). When the nonlinearity is small (i.e., close to linear), the parameter estimates are virtually without bias, normally distributed, and have minimum variance; therefore, any inferences based on them are valid (Ratkowsky 1983, Seber and Wild 1989). To ensure that the standard regression assumptions were met, the residuals were plotted and analyzed as follows:

- A t -test was used to determine if the expected value of the residuals (ε) is significantly different from zero ($H_0: E[\varepsilon] = 0$);
- Shapiro and Wilk's (1965) W statistic was used to test for normally distributed residuals (H_0 : the ε are normally distributed); and
- Endrenyi and Kwong's (1981) F_k test was used to determine if the residuals were homoscedastic (H_0 : the ε have a constant variance across the range of observed growth intercepts).

The sub-models (i.e., model [2] with $A = 1, 2, \dots, 30$) were analyzed independently.

After developing the sub-models, I plotted the parameter values against breast height age (Figure 1) and noticed a strong relationship between them. This suggested that the single equation model given by equation (3) would be suitable for estimating site index from the growth intercept and age.

$$SI = b_1 \cdot e^{b_2 \cdot A} \cdot GI_A^{b_3 \cdot e^{b_4 \cdot A}} + \varepsilon, \quad (3)$$

where: b_1, b_2, b_3, b_4 are model parameters.

This model was fitted with nonlinear least-squares regression. The relationship between parameter $b_{A,1}$ in model (1) and

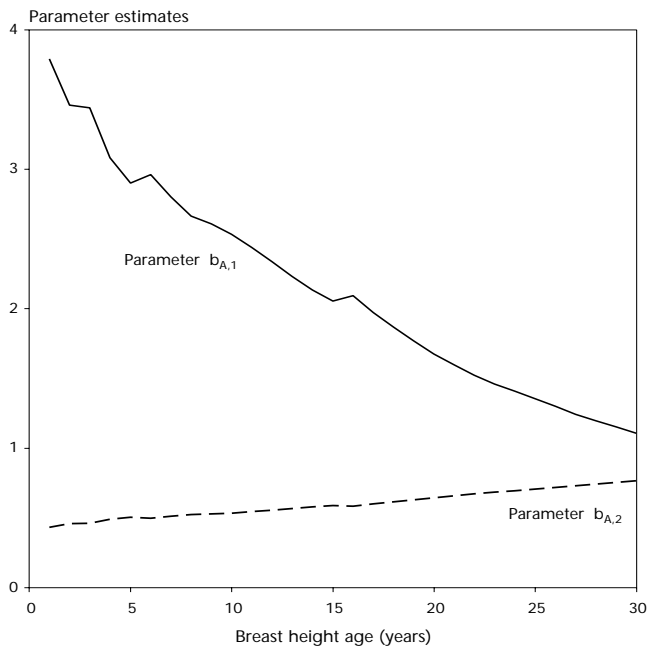


FIGURE 1 *Estimates of parameters $b_{A,1}$ and $b_{A,2}$ versus breast height age.*

breast height age appears to be linear (Figure 1); however, an exponential form of this parameter in model (3) fit slightly better. An analysis of the residuals was not done because the lack of independence between the observations meant that the variance estimates were unreliable. However, the point estimates of the parameters should not be biased by this lack of independence (Sen and Srivastava 1990).

To test the models the site index was estimated from sub-models that had the corresponding set of parameters ($b'_{A,1}$ and $b_{A,2}$, $A = 1, 2, \dots, 30$) and GI_A from the test data. Then, the mean error and the standard deviation of the error were plotted against breast height age. The single equation growth intercept model (equation [3]) and, for comparison, the updated height-age curves for lodgepole pine were also tested in the same way.

RESULTS

Table 1 presents the analysis results for model (2) with breast height age 1 to 30. It includes the parameter estimates and their standard errors, the root mean square error (RMSE), measures of nonlinearity (intrinsic [IN] and parameter effects [PE] nonlinearity and % bias⁶), and the results of residuals analysis (tests for bias, normality [W], and homoscedasticity [F_k]). Note that the sample size is 45 for each of the sub-models. The regression assumptions were met for each of the sub-models. The measures of nonlinearity and parameter bias were small, which indicated that the parameter estimates were virtually linear (Ratkowsky 1983). Table 2 presents the sub-models in the original form (i.e., as equation [1]).

Table 3 presents the analysis results for the single equation variable growth intercept model (equation [3]). The sample size for this model was 1350 observations. The standard errors are biased by serial correlation. The final formulation of this model is:

$$\widehat{SI} = 3.354 \cdot e^{-0.03226 \cdot A} \cdot GI^{0.4723} \cdot e^{0.01466 \cdot A}$$

where: \widehat{SI} is the estimated site index.

Figures 2 and 3 show the mean error and the standard deviation of the errors for the test data, respectively, plotted against breast height age for the three models (single equation and sub-model forms of the variable growth intercept model and the height-age curves).

6 The tests for intrinsic and parameter effects nonlinearity should be less than $1 \div (2 \cdot \sqrt{F_{p,n-p,\alpha}})$, or ≈ 0.2787 at $\alpha=0.05$, where p is the number of parameters in the model, n is the number of observations, and α is the significance level; the bias should be less than 1% (Ratkowsky 1983).

TABLE 1 Results of the analysis of the variable growth intercept sub-models

Age	Parameter	Estimate	Standard error	Root mean error ²	Measures of nonlinearity ^a			Residuals analysis ^b		
					% Bias	IN	PE	Bias	W	F _k
1	b' _{1,1}	1.333	0.1664	1.938	-0.04213	0.02547	0.03203	-0.01127 (0.97)	0.9650 (0.29)	1.062 (0.46)
	b _{1,2}	0.4338	0.04340		0.02052					
2	b' _{2,1}	1.241	0.1740	1.910	-0.06947	0.02309	0.03380	-0.00965 (0.97)	0.9667 (0.33)	1.200 (0.38)
	b _{2,2}	0.4592	0.04555		0.03702					
3	b' _{3,1}	1.235	0.1794	1.942	-0.08108	0.02331	0.03517	-0.00912 (0.97)	0.9638 (0.26)	1.220 (0.37)
	b _{3,2}	0.4615	0.04706		0.04444					
4	b' _{4,1}	1.126	0.1632	1.726	-0.08483	0.02026	0.03262	-0.00538 (0.98)	0.9646 (0.28)	1.211 (0.37)
	b _{4,2}	0.4898	0.04270		0.04179					
5	b' _{5,1}	1.065	0.1553	1.619	-0.07789	0.02041	0.03039	-0.00513 (0.98)	0.9690 (0.39)	1.295 (0.33)
	b _{5,2}	0.5054	0.04059		0.03504					
6	b' _{6,1}	1.086	0.1457	1.564	-0.04744	0.02324	0.02697	-0.00853 (0.97)	0.9757 (0.61)	1.110 (0.43)
	b _{6,2}	0.4989	0.03797		0.01945					
7	b' _{7,1}	1.030	0.1419	1.493	-0.04864	0.02173	0.02598	-0.00824 (0.97)	0.9782 (0.70)	1.122 (0.42)
	b _{7,2}	0.5127	0.03691		0.01883					
8	b' _{8,1}	0.9793	0.1413	1.455	-0.05309	0.02104	0.02566	-0.00727 (0.97)	0.9816 (0.81)	1.056 (0.46)
	b _{8,2}	0.5245	0.03665		0.01962					

TABLE 1 (Continued)

Age	Parameter	Estimate	Standard error	Root mean error ²	Measures of nonlinearity ^a			Residuals analysis ^b		
					% Bias	IN	PE	Bias	W	F _k
9	b' _{9,1}	0.9584	0.1409	1.438	-0.05603	0.02036	0.02562	-0.00662 (0.98)	0.9822 (0.83)	1.153 (0.40)
	b _{9,2}	0.5288	0.03647		0.02040					
10	b' _{10,1}	0.9300	0.1389	1.404	-0.05517	0.01953	0.02499	-0.00669 (0.97)	0.9812 (0.79)	1.206 (0.38)
	b _{10,2}	0.5357	0.03591		0.01923					
11	b' _{11,1}	0.8910	0.1340	1.339	-0.05470	0.01800	0.02400	-0.00648 (0.97)	0.9756 (0.60)	1.671 (0.19)
	b _{11,2}	0.5455	0.03461		0.01815					
12	b' _{12,1}	0.8494	0.1309	1.288	-0.05640	0.01684	0.02334	-0.00615 (0.97)	0.9776 (0.68)	1.686 (0.19)
	b _{12,2}	0.5563	0.03382		0.01777					
13	b' _{13,1}	0.8031	0.1297	1.252	-0.05922	0.01610	0.02287	-0.00578 (0.98)	0.9809 (0.79)	1.775 (0.17)
	b _{13,2}	0.5686	0.03351		0.01748					
14	b' _{14,1}	0.7576	0.1296	1.228	-0.06295	0.01565	0.02257	-0.00563 (0.98)	0.9807 (0.78)	1.744 (0.17)
	b _{14,2}	0.5808	0.03352		0.01733					
15	b' _{15,1}	0.7195	0.1282	1.197	-0.06470	0.01519	0.02208	-0.00568 (0.97)	0.9846 (0.89)	1.657 (0.20)
	b _{15,2}	0.5912	0.03317		0.01674					
16	b' _{16,1}	0.7385	0.1285	1.211	-0.05712	0.01608	0.02176	-0.00733 (0.97)	0.9777 (0.68)	1.359 (0.30)
	b _{16,2}	0.5861	0.03324		0.01486					

TABLE 1 (Continued)

Age	Parameter	Estimate	Standard error	Root mean error ²	Measures of nonlinearity ^a			Residuals analysis ^b		
					% Bias	IN	PE	Bias	W	F _k
17	b' _{17,1}	0.6797	0.1276	1.175	-0.06043	0.01555	0.02119	-0.00749	0.9709	1.225
	b _{17,2}	0.6020	0.03305		0.01422			(0.97)	(0.45)	(0.37)
18	b' _{18,1}	0.6245	0.1273	1.149	-0.06433	0.01529	0.02074	-0.00754	0.9587	1.162
	b _{18,2}	0.6170	0.03302		0.01366			(0.96)	(0.17)	(0.40)
19	b' _{19,1}	0.5706	0.1277	1.127	-0.07139	0.01507	0.02051	-0.00729	0.9506	1.099
	b _{19,2}	0.6317	0.03317		0.01370			(0.97)	(0.08)	(0.44)
20	b' _{20,1}	0.5157	0.1282	1.106	-0.08012	0.01492	0.02029	-0.00698	0.9498	1.064
	b _{20,2}	0.6467	0.03332		0.01374			(0.97)	(0.08)	(0.46)
21	b' _{21,1}	0.4680	0.1280	1.085	-0.08781	0.01478	0.01998	-0.00681	0.9488	1.005
	b _{21,2}	0.6601	0.03331		0.01350			(0.97)	(0.07)	(0.50)
22	b' _{22,1}	0.4201	0.1267	1.056	-0.09590	0.01448	0.01955	-0.00658	0.9493	1.013
	b _{22,2}	0.6738	0.03305		0.01308			(0.97)	(0.07)	(0.49)
23	b' _{23,1}	0.3783	0.1260	1.034	-0.1040	0.01422	0.01918	-0.00648	0.9507	1.026
	b _{23,2}	0.6859	0.03290		0.01261			(0.97)	(0.08)	(0.48)
24	b' _{24,1}	0.3411	0.1244	1.009	-0.1114	0.01390	0.01873	-0.00642	0.9536	1.044
	b _{24,2}	0.6968	0.03255		0.01204			(0.97)	(0.11)	(0.47)

TABLE 1 (Concluded)

Age	Parameter	Estimate	Standard error	Root mean error ²	Measures of nonlinearity ^a			Residuals analysis ^b		
					% Bias	IN	PE	Bias	W	F _k
25	b' _{25,1}	0.3022	0.1225	0.9809	-0.1216	0.01348	0.01827	-0.00624 (0.97)	0.9559 (0.13)	1.790 (0.16)
	b _{25,2}	0.7083	0.03210		0.01153					
26	b' _{26,1}	0.2616	0.1195	0.9444	-0.1344	0.01295	0.01768	-0.00605 (0.97)	0.9602 (0.20)	1.697 (0.19)
	b _{26,2}	0.7203	0.03137		0.01092					
27	b' _{27,1}	0.2168	0.1177	0.9162	-0.1580	0.01254	0.01725	-0.00577 (0.97)	0.9637 (0.26)	1.549 (0.23)
	b _{27,2}	0.7334	0.03096		0.01053					
28	b' _{28,1}	0.1782	0.1149	0.8832	-0.1849	0.01208	0.01674	-0.00547 (0.97)	0.9668 (0.33)	1.552 (0.23)
	b _{28,2}	0.7450	0.03027		0.01005					
29	b' _{29,1}	0.1418	0.1119	0.8501	-0.2228	0.01162	0.01622	-0.00519 (0.97)	0.9686 (0.38)	1.541 (0.23)
	b _{29,2}	0.7560	0.02953		0.009563					
30	b' _{30,1}	0.1008	0.1094	0.8205	-0.3012	0.01120	0.01574	-0.00497 (0.97)	0.9725 (0.50)	1.535 (0.23)
	b _{30,2}	0.7682	0.02894		0.009103					

^a Includes intrinsic (IN) and parameter effects (PE) nonlinearity.

^b The p-values associated with these tests are in brackets below the value of the test statistics.

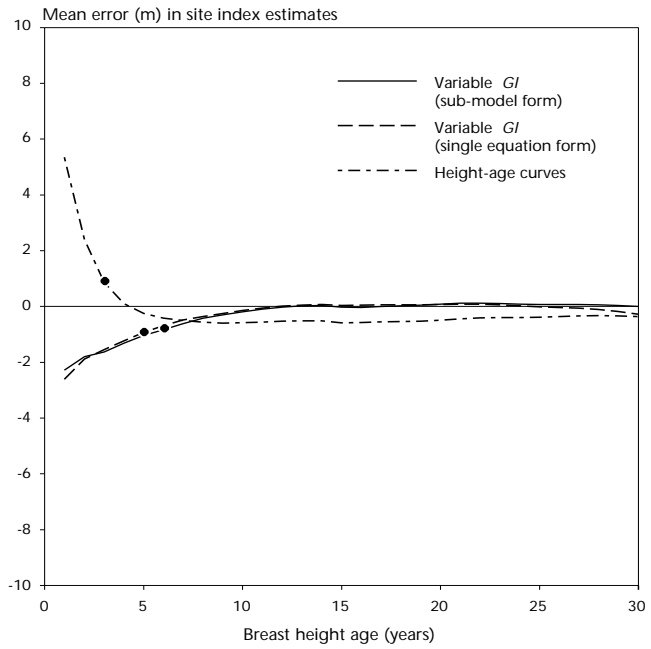


FIGURE 2 Mean error of the single equation and sub-model forms of the variable growth intercept model and the height-age curves. The dots indicate where the mean error becomes not statistically different from zero.

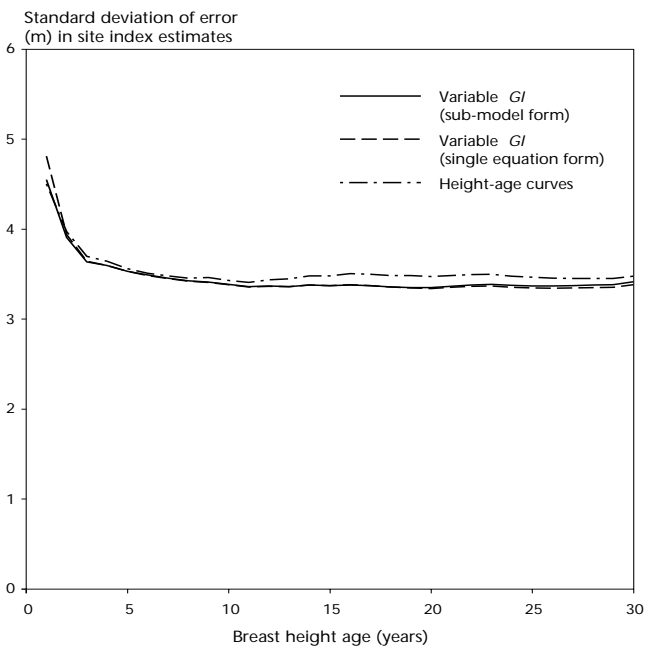


FIGURE 3 *Standard deviation of the errors from the single equation and sub-model forms of the variable growth intercept models and the height-age curves.*

TABLE 2 *Sub-models relating the growth intercept to site index for breast height ages 1 to 30*

Age	Sub-model	Age	Sub-model
1	$\widehat{SI} = 3.791 \cdot GI_1^{0.4338}$	16	$\widehat{SI} = 2.093 \cdot GI_{16}^{0.5861}$
2	$\widehat{SI} = 3.460 \cdot GI_2^{0.4592}$	17	$\widehat{SI} = 1.973 \cdot GI_{17}^{0.6020}$
3	$\widehat{SI} = 3.440 \cdot GI_3^{0.4615}$	18	$\widehat{SI} = 1.867 \cdot GI_{18}^{0.6170}$
4	$\widehat{SI} = 3.083 \cdot GI_4^{0.4898}$	19	$\widehat{SI} = 1.769 \cdot GI_{19}^{0.6317}$
5	$\widehat{SI} = 2.902 \cdot GI_5^{0.5054}$	20	$\widehat{SI} = 1.675 \cdot GI_{20}^{0.6467}$
6	$\widehat{SI} = 2.962 \cdot GI_6^{0.4989}$	21	$\widehat{SI} = 1.597 \cdot GI_{21}^{0.6601}$
7	$\widehat{SI} = 2.802 \cdot GI_7^{0.5127}$	22	$\widehat{SI} = 1.522 \cdot GI_{22}^{0.6738}$
8	$\widehat{SI} = 2.663 \cdot GI_8^{0.5245}$	23	$\widehat{SI} = 1.460 \cdot GI_{23}^{0.6859}$
9	$\widehat{SI} = 2.608 \cdot GI_9^{0.5288}$	24	$\widehat{SI} = 1.407 \cdot GI_{24}^{0.6968}$
10	$\widehat{SI} = 2.534 \cdot GI_{10}^{0.5357}$	25	$\widehat{SI} = 1.353 \cdot GI_{25}^{0.7083}$
11	$\widehat{SI} = 2.438 \cdot GI_{11}^{0.5455}$	26	$\widehat{SI} = 1.299 \cdot GI_{26}^{0.7203}$
12	$\widehat{SI} = 2.338 \cdot GI_{12}^{0.5563}$	27	$\widehat{SI} = 1.242 \cdot GI_{27}^{0.7334}$
13	$\widehat{SI} = 2.232 \cdot GI_{13}^{0.5686}$	28	$\widehat{SI} = 1.195 \cdot GI_{28}^{0.7450}$
14	$\widehat{SI} = 2.133 \cdot GI_{14}^{0.5808}$	29	$\widehat{SI} = 1.152 \cdot GI_{29}^{0.7560}$
15	$\widehat{SI} = 2.053 \cdot GI_{15}^{0.5912}$	30	$\widehat{SI} = 1.106 \cdot GI_{30}^{0.7682}$

\widehat{SI} = estimated site index
 GI = growth intercept

TABLE 3 *Results of the analysis of the single equation growth intercept model*

Parameter	Parameter estimate	Standard error	Root mean square error
			1.291
b_1	3.354	0.1040	
b_2	-0.03226	0.001650	
b_3	0.4723	0.007421	
b_4	0.01466	0.0006997	

DISCUSSION

In principle, growth intercept models are a subset of site-index curves, which express site index as a function of height and age. However, in growth intercept models, the age is fixed and the height has undergone a transformation. The single equation form of the variable growth intercept model is closer to a site-

index curve because the age is not fixed, only the height has undergone a simple transformation. The growth intercept models were developed for a much smaller range of ages and are therefore likely to be more accurate than site-index curves. However, this cannot be tested because site-index curves for lodgepole pine in British Columbia have not been published.

The fixed growth intercept model is:

$$\widehat{SI} = -0.1065 + 2.898 \cdot \sqrt{GI}, \quad (4)$$

where: GI is the five-year growth intercept (cm/yr) measured from annual whorls.

The sub-model for breast height age five is:

$$\widehat{SI} = 2.902 \cdot GI_5^{0.5054}.$$

These two models are directly comparable because both estimate site index from the first five years growth above breast height. But while both were developed with data from the same trees and plots, there could be a difference in the accuracy of the data. For instance, the growth intercepts for the fixed growth intercept model were measured accurately after the stem was split to identify the annual whorls. The height-age data for the variable growth intercept models, however, were obtained by interpolation. The two models are essentially identical, which suggests that a high degree of confidence can be placed in the stem analysis data for re-creating early height growth patterns and, consequently, the variable growth intercept model. It also suggests that the practice of splitting the stem to identify annual whorls may be unnecessary. The root mean squared error of the fixed growth intercept model is smaller than that of the variable growth intercept model (1.43 vs 1.62, from Table 1 and J.S. Thrower and Associates Ltd. 1994 [see footnote 3]) because the height growth was measured, rather than estimated, for the fixed formulation.

From an operational standpoint, there are several good reasons for preferring variable over fixed growth intercept models.

- Annual whorls do not need to be identified to determine age. This is a source of error for species such as lodgepole pine that have false whorls.
- No special training is required for the tree measurements, as commonly collected data (age and height) are used.

- The models can be used as soon as one year after the trees reach breast height (a 5-year waiting period is usually required after this height is reached). Therefore, growth intercept sampling can be added to other surveys without modifying the survey schedule.
- For older stands (>5 years breast height age), site index is estimated from all available growth, resulting in more accurate estimates. The fixed growth intercept model does not use this information.
- The same data are required to estimate site index as for height-age curves. Therefore, deficiencies in the height-age curves can be corrected at young (<30 years) ages. The variable model can be programmed into the site index estimation program *Freddie*,⁷ and its use would be transparent.

7 K. Polsson. 1994.
Freddie — site
 index estimation
 program
 documentation for
 version 2.41. Res.
 Br., B.C. Min. For.
 33 p.

Variable growth intercept models do have certain disadvantages as well:

- They require height and age measurements that, on small trees, are subject to large relative measurement errors.
- They cannot be used easily in stands older than 30 years breast height age.
- There may be a loss of precision compared to the fixed growth intercept models.
- Thirty sub-models are required. This can be unwieldy unless data are presented in a tabular (Table 4), computer, or single equation form.

Foresters will want to know the age at which the height-age curves become more accurate than the variable growth intercept models. To find this age, I compared the mean error (observed *SI* – estimated *SI*) (Figure 2) and the standard deviation of the errors (Figure 3) obtained when the test data were applied to both the variable growth intercept models (the sub-model and single equation forms) and to the lodgepole pine height-age curves. As shown in Figure 2, the variable growth intercept models have smaller estimation errors than the height-age curves. The variable growth intercept models overestimated site index until about breast height age five. The height-age curves were considerably biased at young ages and consistently over-estimated site index at older ages. As shown in Figure 3, the variability in the error decreased with age but is

TABLE 4 Estimated site indices from the sub-model form of the variable growth intercept model

bh ring count ^a	Top Height (m)																																				
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28										
1	32																																				
2	20	30	38																																		
3	16	24	30	35	39																																
4	13	21	26	30	34	37	40																														
5	12	18	23	27	30	34	36	39																													
6	11	16	21	24	27	30	33	35	37	39																											
7	9	15	19	22	25	28	30	32	34	36	38	40																									
8	9	14	17	21	23	26	28	30	32	34	36	38	39																								
9	8	13	16	19	22	24	26	28	30	32	34	35	37	38	40																						
10	7	12	15	18	20	23	25	27	28	30	32	33	35	36	38	39	40																				
11	7	11	14	17	19	22	24	25	27	29	30	32	33	35	36	37	39	40																			
12	6	10	14	16	18	21	22	24	26	28	29	31	32	33	35	36	37	39	40																		
13	6	10	13	15	18	20	21	23	25	27	28	29	31	32	34	35	36	37	38	40																	
14	6	9	12	15	17	19	21	22	24	26	27	28	30	31	33	34	35	36	37	39	40																
15	5	9	12	14	16	18	20	21	23	25	26	28	29	30	31	33	34	35	36	37	39	40															
16	5	9	11	13	15	17	19	21	22	24	25	26	28	29	30	31	33	34	35	36	37	39	40														
17	5	8	11	13	15	17	18	20	21	23	24	26	27	28	29	31	32	33	34	35	36	37	38	39	40												
18	4	8	10	12	14	16	18	19	21	22	24	25	26	28	29	30	31	32	33	34	35	36	37	38	39	40											
19	4	7	10	12	14	15	17	19	20	22	23	24	26	27	28	29	30	32	33	34	35	36	37	38	39	40											
20	4	7	9	11	13	15	16	18	20	21	22	24	25	26	27	29	30	31	32	33	34	35	36	37	38	39	40										
21	4	6	9	11	13	14	16	17	19	20	22	23	24	26	27	28	29	30	31	33	34	35	36	37	38	39	40										
22	3	6	8	10	12	14	15	17	18	20	21	22	24	25	26	27	29	30	31	32	33	34	35	36	37	38	39	40									
23	3	6	8	10	12	13	15	16	18	19	21	22	23	24	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40								
24	3	6	8	10	11	13	15	16	17	19	20	21	23	24	25	26	27	29	30	31	32	33	34	35	36	37	38	39	40								
25	3	5	7	9	11	13	14	16	17	18	20	21	22	23	25	26	27	28	29	30	31	32	33	34	36	37	38	39	40								
26	3	5	7	9	11	12	14	15	17	18	20	21	22	23	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40							
27	3	5	7	9	10	12	13	15	16	17	19	20	21	22	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40						
28	2	5	7	8	10	11	13	14	16	17	18	20	21	22	23	24	25	27	28	29	30	31	32	33	34	35	36	37	38	39	40						
29	2	4	6	8	10	11	13	14	15	17	18	19	20	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40					
30	2	4	6	8	9	11	12	14	15	16	17	19	20	21	22	23	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40					

^a The breast height (bh) age of the tree is one-half less than the ring count at breast height.

similar for all three models. Because the test data set is actually a subset of the data used to update the height-age curves, the errors for the height-age curves are probably less than they would have been had the data been independent of the curves. Nevertheless, the growth intercept models are still more accurate. However, the sub-models may not be reliable below three years breast height age for two reasons.

1. Variable growth intercept models have excessive bias and variability at these ages (see Figures 2 and 3).
2. Sub-models were developed from the accurate internode measurements for breast height ages one to five and were compared to those developed from the interpolated data. The sub-models for ages one and two gave substantially different estimated site indices.

Based on these analyses, I conclude that the variable growth intercept models can be used in stands from breast height age 3 to 30. Model accuracy increases as the age of the stand increases, and this should be considered when a variable growth intercept survey is scheduled. The models can be used in stands younger than three years breast height age but the estimates of site index may be unreliable. In stands older than 30 years breast height age, site index should be estimated from the height-age curves.

This research now provides up to 31 models to estimate site index — up to 30 sub-models associated with the variable growth intercepts, and the height-age curves. This raises the question of how consistent the site-index estimates are between the models. The transitions between the variable growth intercept sub-models and the height-age curves are shown in Figure 4, as a plot of estimated site index versus age. Height is fixed at 5, 10, 15, 20, and 25 m. Up to age 30, the sub-models estimate site index; afterwards, site index is estimated from the height-age curves. The transitions between the sub-models are smooth (smooth curves mean smooth transitions), as are the transitions to the height-age curves at site indices below approximately 20 m. At higher site indices, the discontinuity with the height-age curve increases but for most applications this increase can be considered minor.

The variable growth intercept model is suitable for use in silviculture surveys. The procedure can be applied to stands between 3 and 30 years breast height age. The top height trees would be identified according to the current standard (for example, Forest Productivity Councils of British Columbia

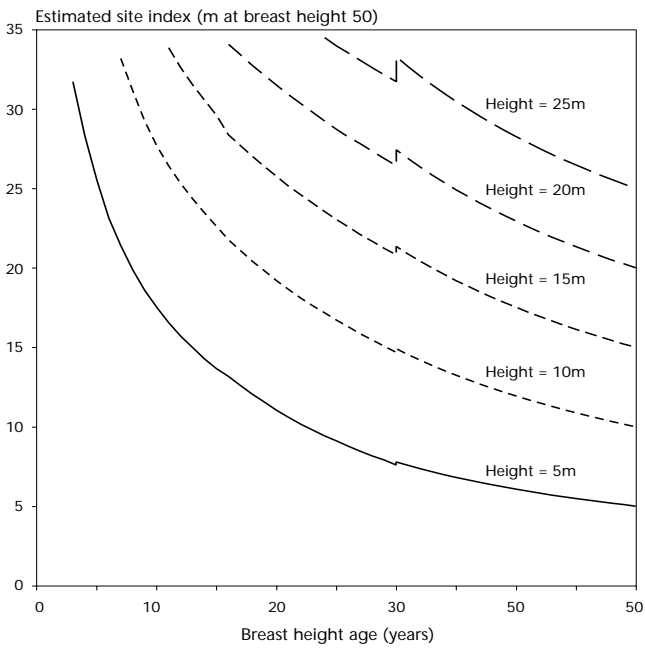


FIGURE 4 Transition between the variable growth intercept sub-models and the height-age curves.

1993) and total height and breast height age measurements would be taken on these trees. Height is measured with a height pole, clinometer, or by felling the tree and measuring its length. Age is determined by extracting a core with an increment borer at breast height, or by felling the tree and counting the rings at breast height. If more than one tree is measured in a plot, the site index of each tree should be determined and then averaged to obtain a site index for the plot. An alternative method is to use averaged heights and ages to estimate site index. However, two minor problems are encountered when site indices are estimated in this way. First, the averaged age is likely not a whole number so the question of which sub-model to use is raised. This problem does not exist with the single equation formulation. Second, a small error may be introduced because different sub-models are used for different ages.

The calculation of breast height age from the ring count can also be problematic because the internodes do not always begin at breast height. Two situations can occur:

1. The growth intercept is measured during the dormant season (Figure 5). On average, breast height lies midway between nodes so the age of the tree is one-half year less than the ring count, n . Therefore, the age of the tree is $(n - 0.5)$.
2. The growth intercept is measured during the growing season. In this case, the height and ring count that correspond to the end of the last growing season should be measured and the age of the tree is again $(n - 0.5)$.

In both situations, site index should be estimated with the sub-model corresponding to age n , where n is the number of complete rings. An argument could be made for using the sub-model corresponding to age $(n - 1)$. However, my analysis shows that the estimated site index will differ by less than 0.6 m (usually much less, except for extremely high sites) between the two sub-models.

To avoid this complication, another method to determine the growth intercept is to assume that breast height age zero starts at the first node above breast height. Age and height are then determined by counting nodes and measuring internodal distances. Although this method is not consistent with the way the models were developed, it should be acceptable because the average height growth at the first internode above breast height should be similar to the average height growth at breast height.

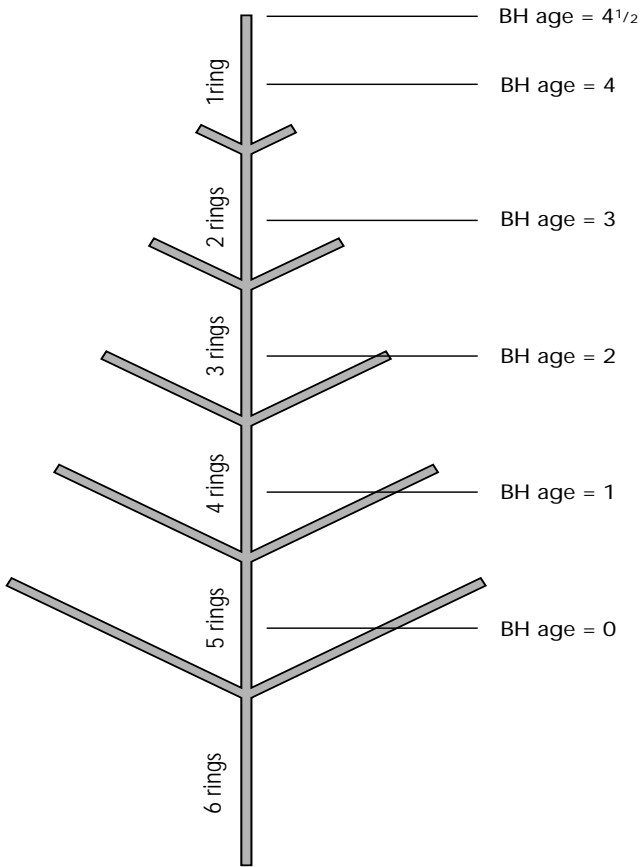


FIGURE 5 *Stylized tree showing the age calculation. The number of annual growth rings in each internode are indicated, as well as the height attained by the tree at various breast height ages. It is assumed that, on average, breast height lies midway between the branch nodes.*

CONCLUSION

A new concept for growth intercept modelling is presented in this report. It is referred to as a variable growth intercept model because the time period over which the growth intercept is measured is not fixed. The model is flexible and improves our ability to accurately determine site index in juvenile lodgepole pine stands. The models can be used in silviculture surveys because they are applicable to young stands.

I recommend the sub-model form of the variable growth intercept model over the single equation form for estimating site index in juvenile stands of lodgepole pine because of its slightly better accuracy and superior statistical properties. However, the two forms give such similar estimates of site index that the difference between them is negligible. I recommend using the models in stands between 3 and 30 years breast height age. Site index for stands older than 30 years should be estimated from the height-age curves.

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