RESEARCH REPORT

01

Site Index Conversion Equations for Mixed Species Stands

Gordon D. Nigh

1995

Province of British Columbia
Ministry of Forests Research Program

Site Index Conversion Equations for Mixed Species Stands

Gordon D. Nigh

Province of British Columbia
Ministry of Forests Research Program

Canadian Cataloguing in Publication Data

Nigh, Gordon Donald, 1956-
Site index conversion equations for mixed species stands

(Research report ; 01)

Includes bibliographical references: p.
ISBN 0-7726-2293-0

1. Site index (Forestry) - British Columbia. 2. Pinaceae - Growth. I. British Columbia. Ministry of Forests. Research Branch. II. Title. III. Series: Research report (British Columbia. Ministry of Forests. Research Branch) ; 01.

SD 396.N53 1994 634.9′75′09711 C94-960368-6

Reprinted in part from the *Forest Science* (vol. 41 no. 1), published by the Society of American Foresters, 5400 Grosvenor Lane, Bethesda, MD 20814-2198. Not for further reproduction.

Published by the
Research Branch
Ministry of Forests
31 Bastion Square
Victoria, B.C. V8W 3E7

Copies of this and other Ministry of Forests
titles are available from
Crown Publications Inc.
521 Fort Street
Victoria, B.C. V8W 1E7

ABSTRACT

Models are developed to predict the site index of one species in a mixed stand from the site index of a cohort. These models are applicable to site quality evaluation, growth and yield modelling, and timber management planning. A single-equation compatible site index conversion system is developed using the geometric mean regression (GMR) line for the following species mixes: coastal Douglas-fir/western hemlock; interior Douglas-fir/lodgepole pine, Douglas-fir/white spruce, Douglas-fir/western hemlock, Douglas-fir/western larch, white spruce/lodgepole pine, white spruce/subalpine fir, lodgepole pine/subalpine fir, and lodgepole pine/western larch.

ACKNOWLEDGEMENTS

I would like to thank the Growth and Yield section of the Inventory Branch for their data and co-operation; and Ken Mitchell, Ian Cameron, Jim Goudie, Wendy Bergerud, and Albert Nussbaum of the Research Branch, and Dr. Vic Smith for their helpful comments.

CONTENTS

Abstract iii

Acknowledgements iv

Introduction 1

Data 2

Methods 3

Results and Discussion 5

Conclusions 17

Literature Cited 19

Tables

1. Species mixes for which site index conversion equations were developed **2**
2. Results of the geometric mean regression analysis **15**
3. Formulated site index conversion equations **15**
4. Stratification types for the species mixes **17**

Figures

1. Scatter plot of data showing the fitted GMR relationship between the coastal Douglas-fir and western hemlock site indices **6**
2. Scatter plot of data showing the fitted GMR relationship between the interior Douglas-fir and lodgepole pine site indices **7**
3. Scatter plot of data showing the fitted GMR relationship between the interior Douglas-fir and white spruce site indices **8**
4. Scatter plot of data showing the fitted GMR relationship between the interior Douglas-fir and western hemlock site indices **9**
5. Scatter plot of data showing the fitted GMR relationship between the interior Douglas-fir and western larch site indices **10**

6. Scatter plot of data showing the fitted GMR relationship between the interior white spruce and lodgepole pine site indices 11
7. Scatter plot of data showing the fitted GMR relationship between the interior white spruce and subalpine fir site indices 12
8. Scatter plot of data showing the fitted GMR relationship between the interior lodgepole pine and subalpine fir site indices 13
9. Scatter plot of data showing the fitted GMR relationship between the interior lodgepole pine and western larch site indices 14

INTRODUCTION

Site index is the average height of site trees at a reference age. Trees that are undamaged and experience minimal competition from surrounding trees are selected as site trees because they represent height growth due mainly to site factors (Monserud 1984). Height-age curves[1] show the height development pattern exhibited by site trees of a particular site index throughout their life. They are also used to estimate site index from height and age (García 1983) by iterative numerical techniques.

There are two principal applications for height-age curves. First, they are used to estimate site index (i.e., the potential productivity of a site). Site indices of stands are often compared to a threshold level when making silviculture decisions. Second, height-age curves are used to project the height of a stand through time. Height is an important predictive variable in growth and yield models such as Variable Density Yield Prediction and the Tree and Stand Simulator (Martin 1991). These models are used to determine the long-run sustained yield, set the allowable annual cut, and update forest inventories.

A site index conversion equation is a functional relationship between the site index of two species growing in mixed stands. Although site index and height-age curves have generally been applied to even-aged pure stands, they can be used in mixed stand situations if the site trees are suitable. The height and age of the leading species, from which the site index is calculated, are recorded in the British Columbia forest inventory. A site index conversion equation is valuable in this situation because the site index of the second leading species in a mixed stand is unknown. Conversion equations are also useful in modelling the growth of mixed species stands.

Site index conversion equations are briefly discussed in Carmean (1975), Hägglund (1981), and Vanclay (1992). Specific examples of conversion equations can be found in Doolittle (1958), Foster (1959), Carmean and Vasilevsky (1971), Carmean (1979), and Steele and Cooper (1986). In these studies, simple linear models and conventional least squares describe functional relationships between the site indices of associated species. Two regression equations are needed to make predictions: one to predict the site index of species A from the site index of species B, and the other for the inverse (B from A). However, these lines do not give compatible two-way predictions.[2] That

---

[1] Height-age curves are often referred to as site index curves. However, true site index curves relate site index to height and age which allows direct estimates of site index. True site index curves are not available for many tree species in British Columbia.

[2] Compatible predictions will result if the correlation between the site indices is perfect — a virtually impossible event!

is, if the site index of species A is x and the predicted site index of species B is y, then predicting the site index of species A from y does not give x. Nigh (1995) developed a single-equation, and hence compatible, site index conversion system using the geometric mean regression line. The same approach is used in this research for additional species pairs.

This report presents site index conversion equations developed for some species mixtures (Table 1) in British Columbia.

TABLE 1 *Species mixes for which site index conversion equations were developed*

Coast
Fdc[a] ↔ Hw
Interior
Fdi ↔ Pl
Fdi ↔ Sw
Fdi ↔ Hw
Fdi ↔ Lw
Sw ↔ Pl
Sw ↔ Bl
Pl ↔ Bl
Pl ↔ Lw

[a] Tree species abbreviations:

Bl – Subalpine fir (*Abies lasiocarpa* [Hook.] Nutt.)

Fdc – Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco var. *menziesii*)

Fdi – Douglas-fir (*Pseudotsuga menziesii* var. *glauca* [Beissn.] Franco)

Hw – Western hemlock (*Tsuga heterophylla* (Raf.) Sarg.)

Lw – Western larch (*Larix occidentalis* Nutt.)

Pl – Lodgepole pine (*Pinus contorta* var. *latifolia* Dougl.)

Sw – White spruce (*Picea glauca* [Moench] Voss)

DATA

The data for this study came from temporary inventory sample plots established by the Inventory Branch of the B.C. Ministry of Forests. For each tree in the 0.1 ha plots, the plot identifier, DBH, species, inventory type group (ITG), and forest inventory zone[3] were recorded. Total tree age and height were also recorded for some trees.

Potential plots were screened for suitability to ensure that adequate site trees were available to determine site index. Screening involved the use of four criteria:

1. Heights and ages of the four site trees (two largest diameter trees of each species)[4] are recorded;
2. Range in tree ages is less than or equal to 20 years, following the Forest Productivity Councils of British Columbia definition of even-aged stands (1993);

[3] Forest inventory zone indicates the region where the plot was located (zones A–C are coastal, D–L are interior).

[4] The selection intensity for site trees is less than the recommended 100 trees/ha (i.e., 10 trees in a 0.1 ha plot) and may cause bias. However, this bias should not have any practical effect on the conversion equations because it is proportional for both species (Nigh 1995).

3. Breast height age of both species is between 20 and 150 years, reflecting the age range in which height-age curves are typically assumed to be valid; and,
4. Heights of the site trees are at least 75% of the height of the tallest tree in the plot.

These criteria provide some degree of assurance that the site trees are in the upper stratum of the canopy.

The site index of each species was determined by first averaging the total age and height of the two site trees of each species. Total age was converted to breast height age using functions built into the height-age models (Thrower and Nussbaum 1991; Thrower et al. 1991). Site index was then estimated from the top height, breast height age, and the recommended height-age curves (Thrower and Nussbaum 1991; Thrower et al. 1991).

METHODS

The geometric mean regression (GMR) line, which is a simple linear model, was used to develop a compatible site index conversion system for species in mixed stands (Nigh 1995). The slope of this line is the geometric mean of the slopes generated by regressing y on x and x on y, where y and x are variables related by a simple linear model (in this research, x and y are the site indices of the two species). This line is also known as the standard trend line, the standard (or reduced) major axis of the observations, and the functional mean regression line (Ricker 1984).

The GMR line (equation 1) characterizes the relationships between the site indices of two species growing in mixed stands.

$$SI_i = m \cdot SI_j + b + \varepsilon \qquad [1]$$

where: SI_i = site index of species i,
SI_j = site index of species j,
m = slope parameter,
b = intercept parameter, and
ε = error term.

The choice of which species in a given pair is i or j is arbitrary because the site index of species i can be predicted from the site index of species j (or vice versa) by inverting the equation.

The slope and intercept of the GMR line ($y = m \cdot x + b$) are given by equation 2 (Ricker 1973, 1984). As in linear regression, the line passes through the mean of the x and y variables and from this the intercept can be calculated (Kermack and Haldane 1950; Barker et al. 1988).

$$\hat{m} = \text{sign}\ (S_{xy}) \cdot \sqrt{\frac{S_{yy}}{S_{xx}}} \qquad [2]$$

$$\hat{b} = \bar{y} - \hat{m} \cdot \bar{x}$$

$$\text{where: } S_{yy} = \sum_{i=1}^{n} (y_i - \bar{y})^2,$$

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2,$$

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x}) \cdot (y_i - \bar{y}),$$

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i,$$

$$\bar{y} = \frac{1}{n} \cdot \sum_{i=1}^{n} y_i,$$

sign (S_{xy}) = -1 if $S_{xy} < 0$, $+1$ otherwise,

n = number of observations, and

$\hat{m}$, $\hat{b}$ are estimators of m and b.

A 95% confidence interval for the slope parameter is given by equation 3 (Ricker 1984).

$$\hat{m} \cdot (\sqrt{B+1} \pm \sqrt{B}) \qquad [3]$$

where: $B = t^2 \cdot (1 - r^2)/(n - 2)$
t = Student statistic for $n - 2$ degrees of freedom at the 95% confidence level,
n = number of observations, and
r = correlation coefficient between x and y.

Geometric mean regression has three desirable properties (Ricker 1984):

1. If the x and y axes are interchanged, the slope is replaced by its reciprocal and the line remains stationary about the data points;
2. The line does not change with a change of scale; and
3. It is robust (i.e., the GMR line usually describes the central trend even when the population and/or the sample are not bivariate normal).

Property (1) is particularly important because the parameters of the GMR line $x = m' \cdot y + b'$ can be derived algebraically from the line $y = m \cdot x + b$ or they can be estimated from the data; either method results in the same line. Therefore, unlike separate linear regressions, consistent estimates of site index will be obtained regardless of which site index is used as the predictive or the predicted variable.

RESULTS AND DISCUSSION

Results of the geometric mean regression analysis are presented in Table 2. The correlation coefficient (r) in Table 2 represents the strength of the relationship as well as the sign of the slope parameter (Ricker 1975). Table 3 shows the complete set of site index conversion equations. Figures 1 through 9 are scatter plots of the data for the species mixes showing the fitted GMR relationship between the species. These scatter plots indicate that the relationships between site indices for these species mixtures are linear, making the GMR line an appropriate model.

Mixtures of species forming the upper strata of a canopy in an even-aged stand can occur in at least three ways (I. Cameron, pers. comm., 1993). The first case, uniform stratification, occurs when both species have been in the upper stratum from stand initiation to stand collapse. In this situation, the predicted site index can be used as a measure of site productivity

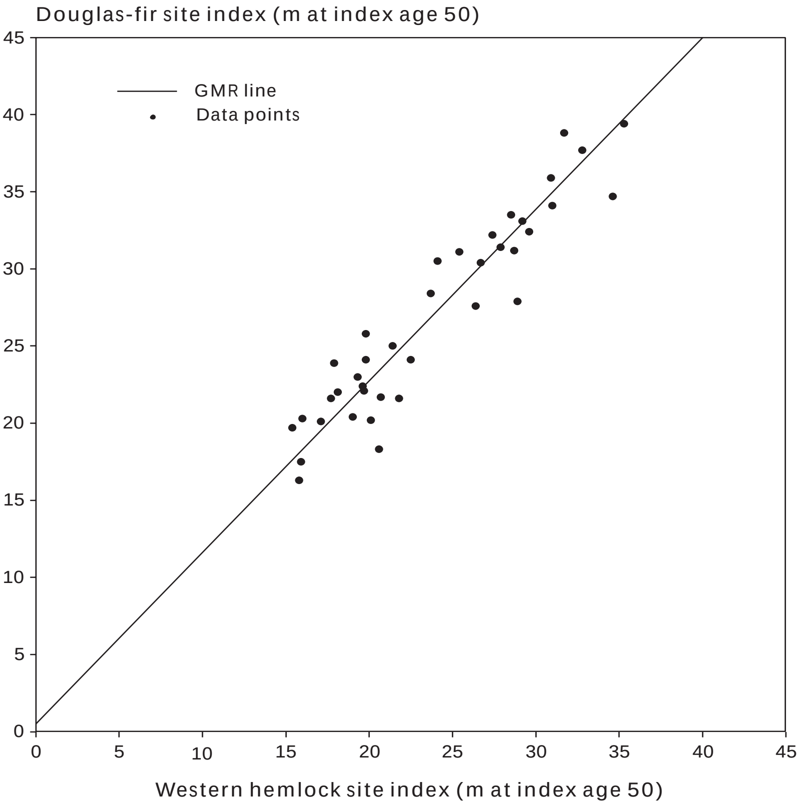

FIGURE 1 *Scatter plot of data (38 plots) showing the fitted GMR relationship between the coastal Douglas-fir and western hemlock site indices.*

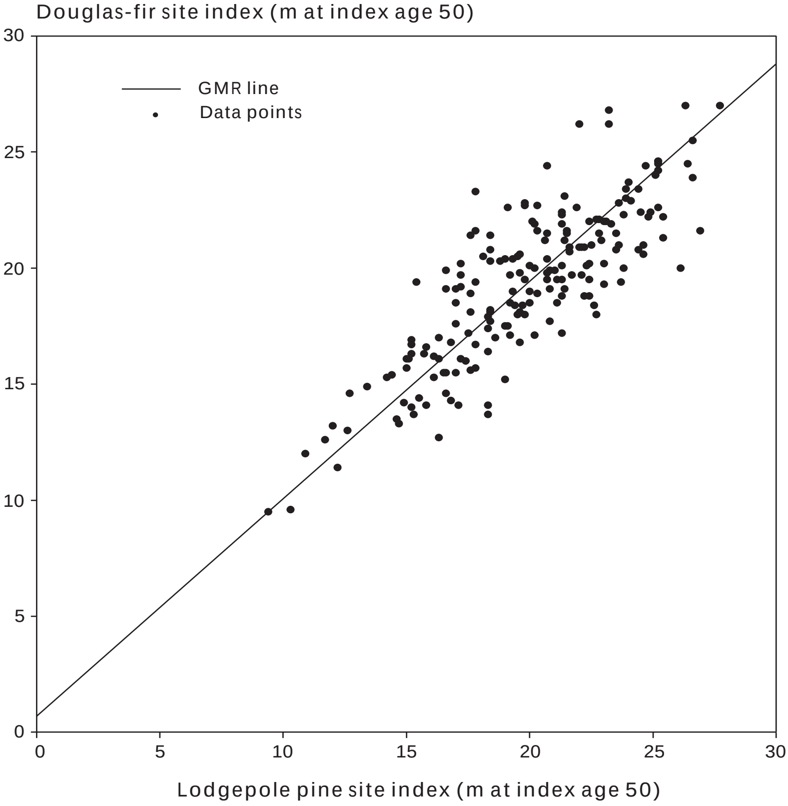

FIGURE 2 *Scatter plot of data (192 plots) showing the fitted GMR relationship between the interior Douglas-fir and lodgepole pine site indices.*

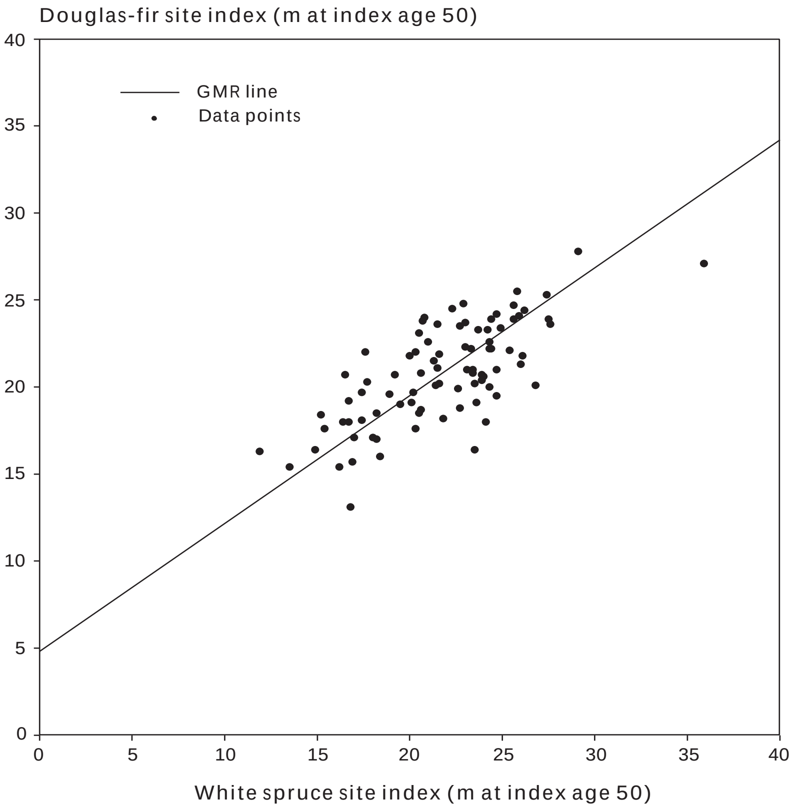

FIGURE 3 *Scatter plot of data (86 plots) showing the fitted GMR relationship between the interior Douglas-fir and white spruce site indices.*

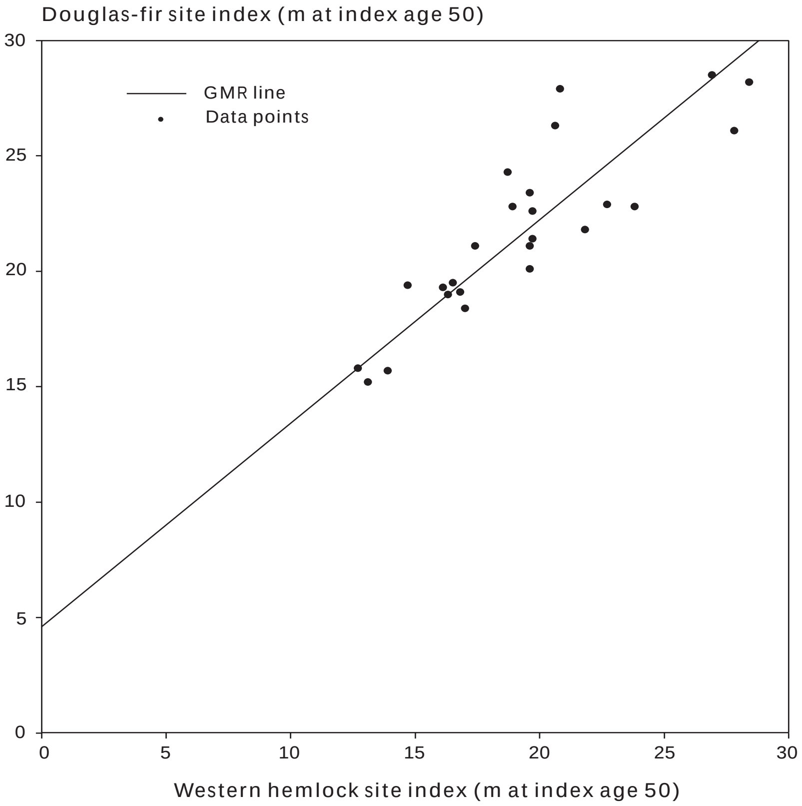

FIGURE 4 *Scatter plot of data (25 plots) showing the fitted GMR relationship between the interior Douglas-fir and western hemlock site indices.*

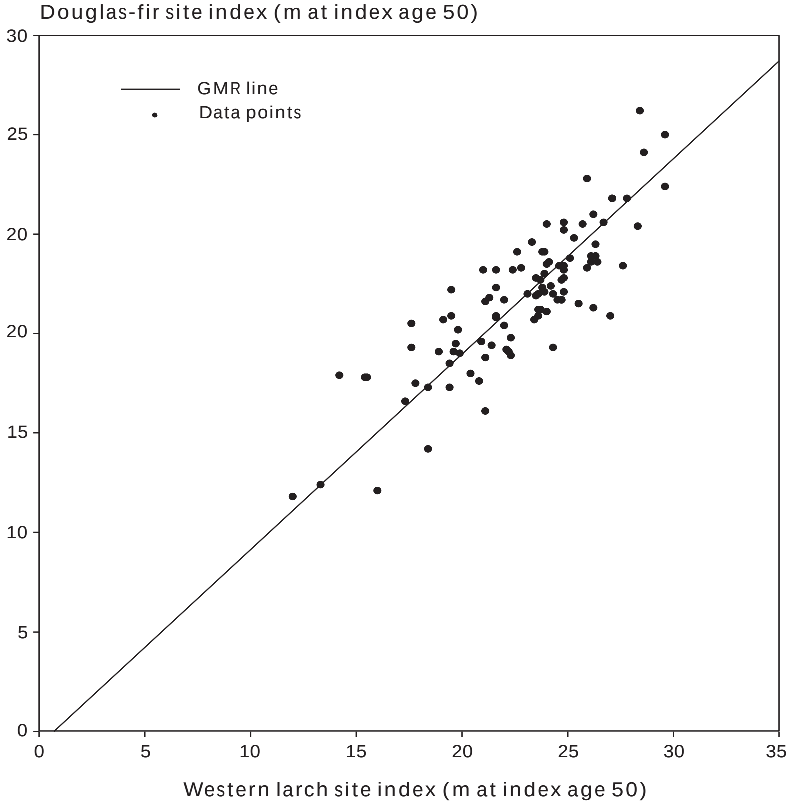

FIGURE 5 *Scatter plot of data (102 plots) showing the fitted GMR relationship between the interior Douglas-fir and western larch site indices.*

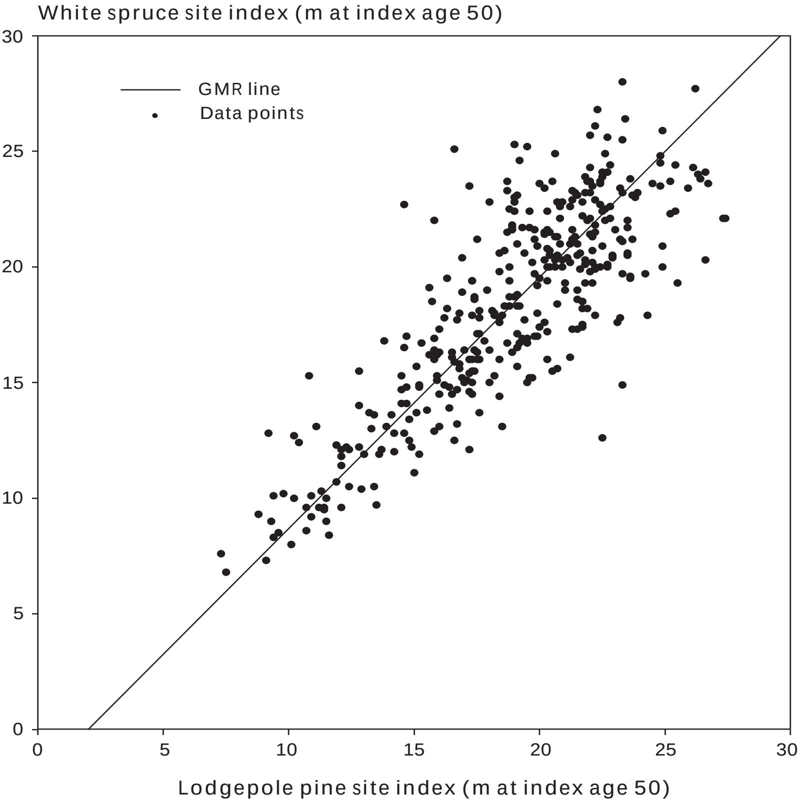

FIGURE 6 *Scatter plot of data (367 plots) showing the fitted GMR relationship between the interior white spruce and lodgepole pine site indices.*

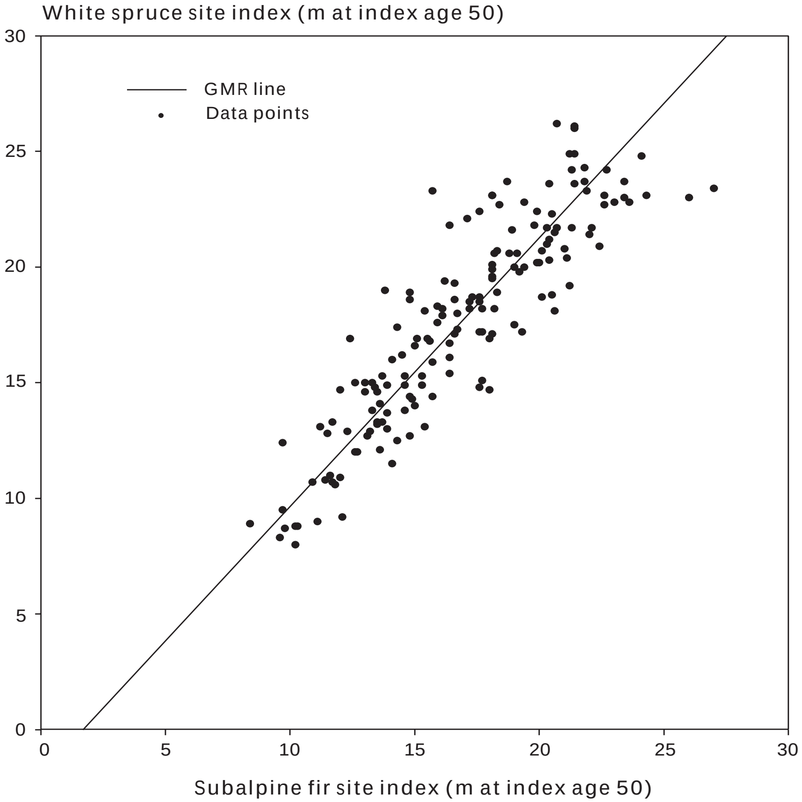

FIGURE 7 *Scatter plot of data (162 plots) showing the fitted GMR relationship between the interior white spruce and subalpine fir site indices.*

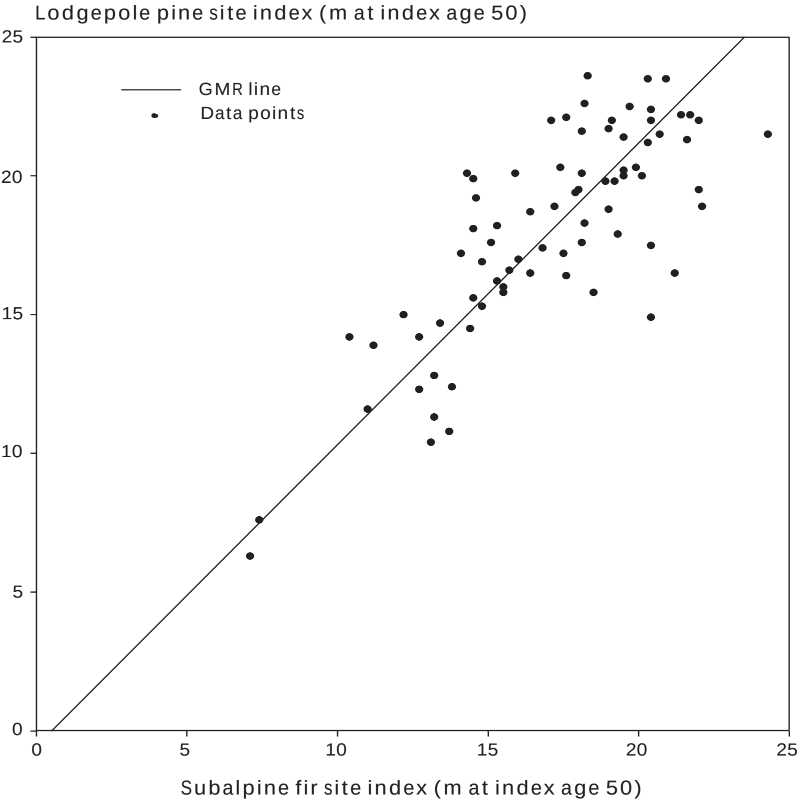

FIGURE 8 *Scatter plot of data (77 plots) showing the fitted GMR relationship between the interior lodgepole pine and subalpine fir site indices.*

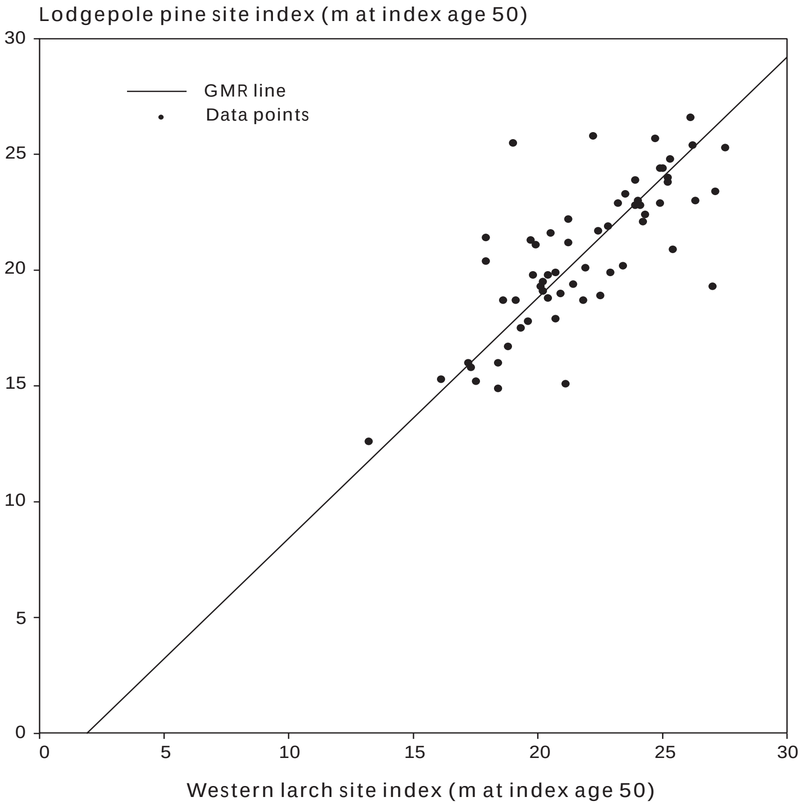

FIGURE 9 *Scatter plot of data (62 plots) showing the fitted GMR relationship between the interior lodgepole pine and western larch site indices.*

TABLE 2 *Results of the geometric mean regression analysis*

Species mix (y/x)	S_{xx}	S_{yy}	r	$\bar{x}$	$\bar{y}$	$\hat{m}$	$\hat{b}$
Coast							
Fdc/Hw	5.7860	6.4357	0.94297	23.71	26.85	1.11	0.480
Interior							
Fdi/Pl	3.5963	3.3632	0.82763	19.87	19.29	0.935	0.709
Fdi/Sw	3.9179	2.8894	0.72538	21.74	20.78	0.737	4.75
Fdi/Hw	4.2351	3.7585	0.86451	19.32	21.71	0.887	4.56
Fdi/Lw	3.4922	3.4345	0.84448	22.83	21.76	0.983	–0.690
Sw/Pl	4.0784	4.4337	0.82638	18.79	18.28	1.09	–2.15
Sw/Bl	3.8149	4.4353	0.88786	16.87	17.66	1.16	–1.95
Pl/Bl	3.4544	3.7563	0.79454	17.01	17.98	1.09	–0.517
Pl/Lw	3.0610	3.1879	0.76487	21.75	20.66	1.04	–2.00

[a] Tree species abbreviations are explained in Table 1.

TABLE 3 *Formulated site index conversion equations*

Coast	
$\mathrm{SI}_{\mathrm{Fdc}}$[a] $= 0.480 + 1.11 \cdot \mathrm{SI}_{\mathrm{Hw}}$	$\mathrm{SI}_{\mathrm{Hw}} = -0.432 + 0.899 \cdot \mathrm{SI}_{\mathrm{Fdc}}$
Interior	
$\mathrm{SI}_{\mathrm{Fdi}} = 0.709 + 0.935 \cdot \mathrm{SI}_{\mathrm{Pl}}$	$\mathrm{SI}_{\mathrm{Pl}} = -0.758 + 1.07 \cdot \mathrm{SI}_{\mathrm{Fdi}}$
$\mathrm{SI}_{\mathrm{Fdi}} = 4.75 + 0.737 \cdot \mathrm{SI}_{\mathrm{Sw}}$	$\mathrm{SI}_{\mathrm{Sw}} = -6.43 + 1.36 \cdot \mathrm{SI}_{\mathrm{Fdi}}$
$\mathrm{SI}_{\mathrm{Fdi}} = 4.56 + 0.887 \cdot \mathrm{SI}_{\mathrm{Hw}}$	$\mathrm{SI}_{\mathrm{Hw}} = -5.14 + 1.13 \cdot \mathrm{SI}_{\mathrm{Fdi}}$
$\mathrm{SI}_{\mathrm{Fdi}} = -0.690 + 0.983 \cdot \mathrm{SI}_{\mathrm{Lw}}$	$\mathrm{SI}_{\mathrm{Lw}} = 0.702 + 1.02 \cdot \mathrm{SI}_{\mathrm{Fdi}}$
$\mathrm{SI}_{\mathrm{Sw}} = -2.15 + 1.09 \cdot \mathrm{SI}_{\mathrm{Pl}}$	$\mathrm{SI}_{\mathrm{Pl}} = 1.97 + 0.920 \cdot \mathrm{SI}_{\mathrm{Sw}}$
$\mathrm{SI}_{\mathrm{Sw}} = -1.95 + 1.16 \cdot \mathrm{SI}_{\mathrm{Bl}}$	$\mathrm{SI}_{\mathrm{Bl}} = 1.68 + 0.860 \cdot \mathrm{SI}_{\mathrm{Sw}}$
$\mathrm{SI}_{\mathrm{Pl}} = -0.517 + 1.09 \cdot \mathrm{SI}_{\mathrm{Bl}}$	$\mathrm{SI}_{\mathrm{Bl}} = 0.475 + 0.920 \cdot \mathrm{SI}_{\mathrm{Pl}}$
$\mathrm{SI}_{\mathrm{Pl}} = -2.00 + 1.04 \cdot \mathrm{SI}_{\mathrm{Lw}}$	$\mathrm{SI}_{\mathrm{Lw}} = 1.92 + 0.960 \cdot \mathrm{SI}_{\mathrm{Pl}}$

[a] Tree species abbreviations are explained in Table 1.

and for height growth modelling because suitable site trees for both species exist. The white spruce/lodgepole pine mixture is an example of uniform stratification. Case two, *consistent stratification,* occurs when the height of one species consistently lags behind the other. For example, in Douglas-fir/western hemlock stands, hemlock is usually shorter than Douglas-fir. Presumably, if hemlock were growing in a pure stand on an identical site, its index would be greater. Site index may not be an appropriate measure of site productivity for hemlock in this situation because the dominant Douglas-fir may be suppressing its height growth. However, this realized site index may be useful in growth modelling when comparing the development of hemlock to that of Douglas-fir in mixed stands. The third class of mixtures, *inconsistent stratification*, occurs when both species may be in the upper strata at some time but not consistently so. In paper birch (*Betula papyrifera* Marsh.)/white spruce stands, the birch may initially overtop the spruce, but as the stand matures the height growth of the birch slows and the spruce eventually overtops the birch. Therefore, for site quality or for growth and yield purposes, it may not be reasonable to apply conversion equations to species exhibiting inconsistent stratification. In fact, the concept of site index may not even apply to these stands.

The type of stratification must be determined when developing the models for a particular species mixture, to ensure that the predicted site indices are used appropriately. To accomplish this, the difference in top height between the two species was plotted against average age. Also, the hypothesis that the difference in top height between the two species is zero was tested. For example, in the spruce/pine stands, both species were about the same height across the range of observed ages. In the Douglas-fir/hemlock stands, however, the fir was consistently 1–4 m taller than the hemlock. The spruce and pine were therefore stratified uniformly and the fir/hemlock stands exhibited consistent stratification. Table 4 lists the types of stratification for the species mixes considered in this research.

The stratification of the target stand must be recognized when applying the models. In any specific mixed species stand, this may differ from that indicated in Table 4, which provides only generalized information. The suitability of the models must be evaluated for each situation. For example, the white spruce/lodgepole pine models were developed for uniformly stratified stands and may not apply to spruce growing under a canopy of pine. In this case, the appropriate model can estimate the site index of the spruce from the site index of the lodgepole pine. However, using this site index for modelling the growth of the spruce without accounting for the pine overstorey would be an inappropriate use of the conversion equations. Furthermore, the site index of the lodgepole pine cannot be reliably predicted from the observed site index of white spruce. Models developed for consistently stratified stands can only be applied to stands of this stratification.

TABLE 4 *Stratification types for the species mixes*

Species mix	Stratification type
Coast	
Douglas-fir/western hemlock	Consistent (Fdc[a] > Hw)
Interior	
Douglas-fir/lodgepole pine	Uniform
Douglas-fir/white spruce	Uniform
Douglas-fir/western hemlock	Consistent (Fdc > Hw)
Douglas-fir/western larch	Consistent (Lw > Fdc)
White spruce/lodgepole pine	Uniform
White spruce/subalpine fir	Consistent (Sw > Bl)
Lodgepole pine/subalpine fir	Uniform
Lodgepole pine/western larch	Consistent (Lw > Pl)

[a] Tree species abbreviations are explained in Table 1.

CONCLUSIONS

The relationship between site indices of species in mixed stands is linear and can be modelled effectively. The geometric mean regression line resolves technical problems associated with linear regression. Stand structure is an important consideration when using site index conversion equations. Three types of stratification were identified for species growing in the upper strata of mixed stands: uniform, consistent, and inconsistent. The type of stratification must be determined when developing the models for a particular species mixture. One must then recognize the stratification of the target stand when applying the models.

LITERATURE CITED

Barker, F., Y.C. Soh, and R.J. Evans. 1988. Properties of the geometric mean functional relationship. Biometrics 44(1):279–281.

Carmean, W.H. 1975. Forest site quality evaluation in the United States. Adv. Agron. 27:209–269.

________. 1979. Site index comparisons among northern hardwoods in northern Wisconsin and upper Michigan. U.S. Dep. Agric. For. Serv. Res. Pap. NC-169. 17 p.

Carmean, W.H. and A. Vasilevsky. 1971. Site-index comparisons for tree species in northern Minnesota. U.S. Dep. Agric. For. Serv. Res. Pap. NC-65. 8 p.

Doolittle, W.T. 1958. Site index comparisons for several forest species in the Southern Appalachians. Proc. Soil Sci. Soc. Amer. 22(5):455–458.

Forest Productivity Councils of British Columbia. 1993. Minimum standards for the establishment and remeasurement of permanent sample plots in British Columbia. Inventory Branch, B.C. Min. For., Victoria, B.C. 22 p.

Foster, R.W. 1959. Relation between site indices of eastern white pine and red maple. For. Sci. 5(3):279–291.

García, O. 1983. A stochastic differential equation model for the height growth of forest stands. Biometrics 39(4):1059–1072.

Hägglund, B. 1981. Evaluation of forest site productivity. For. Abstr. 42(11):515–527.

Kermack, K.A. and J.B.S. Haldane. 1950. Organic correlation and allometry. Biometrika 37(1):30–41.

Martin, P.J. 1991. Growth and yield prediction systems. B.C. Min. For. Spec. Rep. Series 7. 31 p.

Monserud, R.A. 1984. Height growth and site index curves for inland Douglas-fir based on stem analysis data and forest habitat type. For. Sci. 30(4):943–965.

Nigh, G.D. 1995. The geometric mean regression line: a method for developing site index conversion equations for species in mixed stands. For. Sci. 41(1):84–98.

Ricker, W.E. 1973. Linear regressions in fisheries research. J. Fish. Res. Board Can. 30(3):409–434.

________. 1975. A note concerning Professor Jolicoeur's comments. J. Fish. Res. Board Can. 38(8):1494–1498.

________. 1984. Computation and uses of central trend lines. Can. J. Zool. 62(10):1897–1905.

Steele, B.M. and S.V. Cooper. 1986. Predicting site index and height for selected tree species of northern Idaho. U.S. Dep. Agric. For. Serv. Res. Pap. INT-365. 16 p.

Thrower, J.S., A.F. Nussbaum, and C.M. Di Lucca. 1991. Site index curves and tables for British Columbia: interior species. B.C. Min. For., Victoria, B.C. Land Manage. Handb. Field Guide Insert 6.

Thrower, J.S. and A.F. Nussbaum. 1991. Site index curves and tables for British Columbia: coastal species. B.C. Min. For., Victoria, B.C. Land Manage. Handb. Field Guide Insert 3.

Vanclay, J.K. 1992. Assessing site productivity in tropical moist forests: a review. For. Ecol. Manage. 54:257–287.