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## Years to Breast Height and Green-up Age Models Based on a Juvenile Height Model for Lodgepole Pine

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### Abstract

The iterative techniques that are currently used to obtain years to breast height and green-up ages (3 m green-up height) from the juvenile height model for lodgepole pine (*Pinus contorta* var. *latifolia* Dougl.) are cumbersome. Years to breast height and green-up age models were developed by generating age data from the juvenile height model and fitting a model to the data. These models result in years to breast height and green-up age estimates that are consistent with the juvenile height model.

### Introduction

Nigh and Love (1999) constructed a model to estimate the top height of juvenile lodgepole pine (*Pinus contorta* var. *latifolia* Dougl.) trees from site index and total age. This model is valid from total age 0 (germination) to 15. The time it takes to reach a specified top height (e.g., breast height [1.3 m] or green-up height [typically 3 m]) may be obtained from this height model by at least three alternative methods.

1. By iterative techniques; although reliable, in practical terms these techniques require the use of a computer.

2. By inverting the model so that age is expressed as a function of height and site index; however, the formulation of the juvenile height model is such that this is not mathematically possible.
3. By generating age data from the height model and fitting a model to the age data.

Obtaining estimates of years to breast height and green-up age are important. These estimates allow the conversion of heights based on breast height age to heights based on total age, which is necessary for compatibility with growth and yield models. Estimates of green-up age are used in forest management planning to determine when adjacent stands can be harvested. Note that green-up depends on other factors besides height, such as stocking and species composition.

This extension note focuses on the third alternative for obtaining years to breast height and green-up age estimates.

### Methods

Using an iterative technique, years to breast height and green-up age data were generated from the juvenile lodgepole pine height model (equation 1):

$$H = (-0.03993 + 0.004828 \times SI) \times A_t^{1.902} \times 0.9645^{A_t}, \quad [1]$$

where:

- $H$  = top height (m),
- $SI$  = site index (m at breast height age 50), and
- $A_t$  = total age (years).

Site index values from 15 to 30 in steps of one were input into the model; age was varied until the estimated height was 1.3 or 3 m to give years to breast height and green-up ages, respectively. Nonlinear regression was then used to fit a model to the data. The functional form of the model, chosen because it fits the data well, is:

$$YTBH(\text{or } GU) = b_0 \times (SI + b_1)^{b_2} + \varepsilon, \quad [2]$$

where:

- $YTBH$  = years to breast height (years),
- $GU$  = green-up age (years),
- $SI$  = site index (m at breast height age 50),
- $b_0, b_1, b_2$  = model parameters, and
- $\varepsilon$  = the error term.

Different parameter values were estimated for the years to breast height and the green-up age models. The regression assumptions (i.e., the errors are independently and normally distributed with mean 0 and constant variance [Sen and Srivastava 1990]) were tested. To evaluate whether the models' behaviour was linear, tests for intrinsic and parameter effects nonlinearity and bias (Ratkowsky 1983) were performed.

## Results

The years to breast height and green-up age data generated from the juvenile height model are shown in Table 1. The results of the model analyses are presented in Table 2. There was no evidence that the residuals from both models were not normally distributed or homoscedastic. Both models behaved close to linearly based on the criteria for the measures of parameter bias and intrinsic and parameter effects nonlinearity given by Ratkowsky (1983).

Figure 1 presents plots of the fitted

TABLE 1 Years to breast height and green-up age data for site indices from 15 to 30 m

Site index (m @ bha 50)	Years to breast height (years)	Green-up age (years)
15	8.1163	14.1209
16	7.4514	12.8045
17	6.9194	11.7794
18	6.4817	10.9532
19	6.1139	10.2698
20	5.7992	9.6929
21	5.5263	9.1978
22	5.2868	8.7672
23	5.0744	8.3885
24	4.8845	8.0521
25	4.7134	7.7508
26	4.5583	7.4791
27	4.4167	7.7325
28	4.2870	7.0073
29	4.1675	6.8007
30	4.0570	6.6104

TABLE 2 Results of the model analyses

Model	Parameter			$R^2$
	Estimate	Standard error	Root mean squared error (years)	
Years to breast height	$b_0$	21.6623	0.0266793	1.00
	$b_1$	-9.05671	0.00671254	
	$b_2$	-0.550762	0.000363861	
Green-up age	$b_0$	35.9146	0.0491691	1.00
	$b_1$	-9.74731	0.00681460	
	$b_2$	-0.562704	0.000414802	

model overlaid on the data and of the residuals against site index for the years to breast height model. Figure 2 shows the same attributes for the green-up age model. Both models are very good fits to the data. This is expected because the data were gener-

ated from a model that removes the natural variability in years to breast height or green-up age for given levels of site index. The residual plots show a clear trend, indicating that the assumption of independence has been violated. However, the models fit the

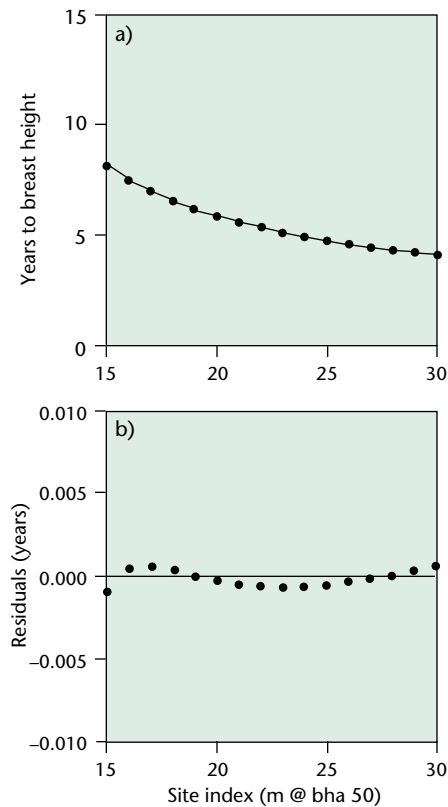


FIGURE 1 Plots of (a) years to breast height (•) and the fitted curve (—) against site index, and (b) residuals (•) against site index.

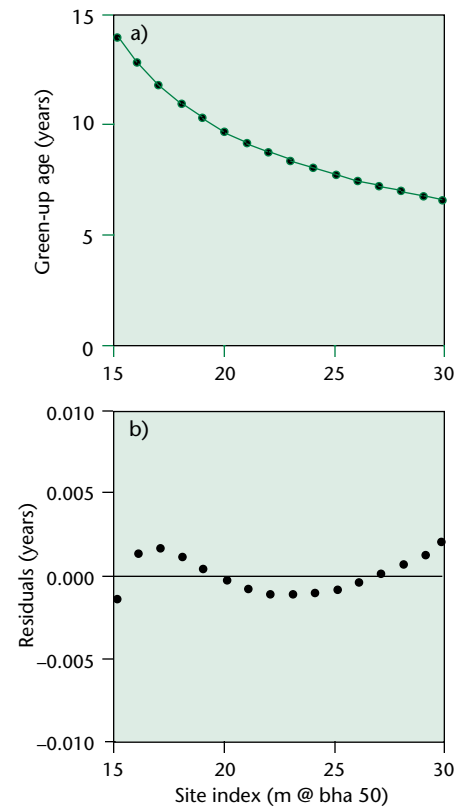


FIGURE 2 Plots of (a) green-up age (•) and the fitted curve (—) against site index, and (b) residuals (•) against site index.

data so well, the residuals are so small, and the parameters are so obviously important and highly significant that further modelling efforts to account for the correlation in the residuals would yield insignificant improvements.

## Discussion

The years to breast height and green-up age models are, respectively:

$$YTBH = 21.6623 \times (SI - 9.05671)^{-0.550762}$$

and,

$$GU = 35.9146 \times (SI - 9.74731)^{-0.562704}.$$

The years to breast height and green-up age models are accurate with respect to the juvenile height model. This can be confirmed from the root mean squared error or by estimating the years to breast height or green-up age from the models and inputting this age into the juvenile height model. The estimated height from the juvenile height model typically differs from 1.3 or 3 m by less than a millimetre.

Green-up height is usually, but not always, 3 m. The technique for generating a green-up age model described in this extension note can be used to develop models for other green-up heights. Given that model (2) performed so well for both years to breast height and 3 m green-up age, it would likely work well for other green-up heights and should be used, at least on the first attempt, in any other green-up age modelling endeavours.

Modelling is usually done with actual data collected in the field or from an experiment. In this study, however, the data were generated from a model, although actual years to breast height and green-up age data could have been obtained from juvenile height data collected in the field. This was not done because the resulting estimates of years to breast height

and green-up age would not have been consistent with the juvenile height model; that is, if the estimated years to breast height or green-up age were input into the juvenile height model, the height model would not have returned a height estimate of 1.3 or 3 m, respectively.

The years to breast height and green-up age data were generated for site indices between 15 and 30 m. The models described in this extension note should not be used for locations with site indices less than 15 m. Also, to avoid potential problems with extrapolation, the models should not be used for locations with site indices outside of the range of the juvenile height model data (18.9–23.3 m). Although the utility of the models can be expanded greatly by extrapolation, model estimates made outside of the site index range of the data may be suspect.

## Conclusion

The models described in this extension note can be used to estimate years to breast height and green-up age (3 m green-up height) for lodgepole pine. These models are consistent with the newly developed juvenile height model for lodgepole pine. The models should be used only for sites with a site index of 15 m or greater.

## References

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