Physically-based estimation of lag time for forested mountainous watersheds

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Abstract This study proposes a method for the estimation of lag time of forested mountainous watersheds. Hydrological flood-flow design methods require a time parameter, such as lag time or time of concentration, to estimate peak discharge and the flood hydrograph shape. Most existing methods used to compute the time parameter of a watershed are empirical and have been developed for urban or rural watersheds. Use of these methods for forested mountainous watersheds may result in severe underestimation of watershed response time and consequently, in overestimation of peak flood discharge. The water flow in a watershed is separated and analysed in two phases, *viz.* the land or hillslope phase and the stream channel phase. Most of the flow in a forested steep watershed is generated through subsurface pathways, and this knowledge, acquired from field experiments, is combined with kinematic wave equations to describe the generation of flow from steep forested hillslopes. This hillslope runoff is then used as input to the stream channels. Kinematic wave equations are developed for the runoff movement in the channels by assuming that both the roughness coefficient and the stream channel slope vary with the distance from the outlet of the watershed. These assumptions are validated and confirmed with data from the USA and Coastal British Columbia, Canada. The resulting equations are integrated to obtain an equation for the lag time. Comparison of the results of the proposed equation with data from two experimental watersheds in Coastal British Columbia and empirical equations used for the calculation of the lag time indicates that the proposed method is reliable and gives a good approximation of the observed lag time. These results are compared with other empirical equation, and it is shown that these earlier methods can result in severe underestimation of lag time.

Estimation physique du temps de réponse de bassins forestiers de montagne

Résumé Cette étude propose une méthode d'estimation du temps de réponse de bassins versants forestiers de montagne. Pour estimer le débit de pointe et la forme de l'hydrogramme, les méthodes d'étude des crues requièrent un paramètre temporel tel que le temps de réponse ou le temps de concentration. La plupart des méthodes actuellement utilisées pour calculer ce paramètre sont empiriques et ont été conçues pour des bassins urbains ou ruraux. L'application de ces méthodes à des bassins boisés montagnards peut conduire à une importante sous-estimation du temps de réponse du bassin et, par voie de conséquence, à une forte surestimation du débit de pointe. Deux phases d'écoulement de l'eau dans un bassin versant ont été prises en considération, la phase terrestre (versant) et la

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phase d'écoulement en rivière. La plus grande partie du débit d'un bassin versant boisé abrupt est produite à travers des chemins de subsurface et cette connaissance, acquise à partir d'expériences de terrain, a été combinée aux équations de l'onde cinématique afin de décrire la production du débit issu des versants forestiers escarpés. Ce débit est alors considéré comme un apport aux chenaux d'écoulement. Les équations de l'onde cinématique sont alors écrites pour décrire l'écoulement dans les chenaux en supposant que le coefficient de rugosité et la pente des chenaux sont fonction de la distance à l'exutoire du bassin. Ces hypothèses ont été validées et confirmées grâce à des données recueillies aux USA et sur le littoral de la Colombie Britannique (Canada). L'intégration de ces équations fournit une équation dont le temps de réponse est l'inconnue. Les résultats de ces calculs ont été confrontés à des observations réalisées sur deux bassins expérimentaux du littoral de la Colombie Britannique. La confrontation montre que la nouvelle méthode est fiable et fournit une bonne approximation du temps de réponse observé. La comparaison de cette nouvelle méthode avec les méthodes empiriques classiques montre que ces dernières peuvent conduire à de graves sous-estimations du temps de réponse.

INTRODUCTION

The determination of peak discharges for a given return period is necessary for the appropriate design of hydraulic structures. Use of the rational method, unit hydrographs and at least some of the commonly used rainfall-runoff models requires estimates of the time of concentration or basin lag time. Various definitions of basin lag time, t_l, and time of concentration, t_c, have been proposed. In this paper, t_l is defined as the time between the centroid of rainfall excess and the hydrograph peak. Time of concentration, t_c, is defined as the travel time for a parcel of water moving from the most remote point in the watershed to the outlet. This time of concentration can be derived from a rainfall hyetograph and the resulting runoff hydrograph by determining the time between the centre of mass of rainfall excess and the inflection point on the recession limb of the direct runoff hydrograph. Both t_l and t_c are measures of watershed response time and are linked by relationships of the type $t_l = at_c$. Overton & Brakensiek (1970) suggested a value equal to 0.6 which was derived by using equilibrium kinematic considerations, whereas Singh (1988) proposed lag time is about 70% of the time of concentration.

Errors in the lag time or time of concentration will cause an error in the design peak flow. Bondelid *et al.* (1982) showed that as much as 75% of the total error in an estimate of the peak discharge can result from errors in the time of concentration. Recognizing the importance of the time parameters in hydrological design, many studies have addressed the problem of estimating the lag time or time of concentration. Most have proposed empirical equations which are the result of regression analysis relating t_l or t_c to the physiographic parameters of the watershed and some to the rainfall parameters, such as the rainfall intensity or duration (Kiprich, 1940; Watt & Chow, 1985; Papadakis & Kazan, 1987; Swenty & Westphall, 1989; Sabol, 1993). Other studies estimate the response time parameters of a watershed by applying simple mathematics based on the kinematic wave approximation of the overland flow (Akan, 1986; Aron *et al.*, 1991). Evaluation of the above equations for regions different from those used for their development showed that most of them are unreliable (Kibler & Aron, 1983; McCuen *et al.*, 1984; Goitom, 1989). In addition, most of the above methods or equations have been developed mainly for small agricultural and urban watersheds where the overland flow is the dominant runoff generation mechanism. There are no methods for the estimation of lag time or time of concentration of forested mountainous watersheds where the storm flow is, usually, generated through subsurface pathways. Because of a lack of proper methods for the estimation of lag time for these mountainous watersheds, hydrologists tend to use methods developed for agricultural regions and application of these methods to mountainous watersheds may result in severe underestimation of the basin response time and consequently, in the overestimation of the peak flow. Such an overestimation of the flood value results in the over-sizing of hydraulic engineering structures and greatly increases the cost of construction.

The objectives of this study are: to develop a physically-based equation for the estimation of lag time of mountainous forested watersheds by using simple mathematics and physical observations; to test the validity of the proposed equation by using data from two experimental watersheds; and to illustrate the errors that could result from the application of several of the well known equations which will be shown to be unsuitable for mountain use and which were developed for agricultural or urban watersheds. Such an analysis is aimed at providing the design engineer with a more reliable method for the estimation of lag time and to indicate the errors associated with the application of empirically derived equations.

DEVELOPMENT OF THE LAG TIME EQUATION

The runoff of rainwater in a watershed can be separated into two components: the runoff in the land phase and the flow in the stream system. In this study separate discussions will be made for the two components of water flow in a watershed.

Land phase

To evaluate the hydrological response of a watershed, certain fundamental knowledge is desirable, for example knowledge of the pathways that water follows on its way from the hillslopes to the stream channel. Three models of runoff generation have been proposed over the years: the Hortonian overland flow model, the partial areas model and the variable source area model. The Hortonian overland flow model (Horton, 1933) suggests that the runoff is generated by overland flow due to large rainfall intensity and the limited capacity of the soil to absorb the rainwater. The partial areas model (Dunne & Black, 1970) proposes that the runoff on a hillslope is generated when saturation of the soil in the riparian areas occurs because of the rising groundwater table and subsequent overland flow over these areas. According to this model, the subsurface flow is only a minor contributor to the storm flow hydrograph because its response is damped by storage and transmission through soils. The third model for runoff generation, the variable source area model, was developed from observations in forest watersheds by Hursh (1944). According to this model the runoff is generated by subsurface flow and direct rainfall to the stream and riparian areas. The mechanisms for producing quick subsurface flow response are: (a) the soil matrix "translatory flow" (Hewlett & Hibbert, 1967) in which the infiltrated water in the hillslopes tends to displace most of the water ahead of it and which was stored during previous rainfall events; and (b) the flow in macropores or "soil pipes" (Jones, 1971) which deliver the infiltrated water rapidly to the stream channel. These "soil pipes" are developed by the action of insects, small animals, tree roots, and chemical weathering and are sustained by frequent water passage.

Overland flow is unlikely to occur in forested hillsides because of the high infiltration capacity of the soil. If overland flow does occur, it will probably be concentrated in the riparian areas. Many researchers (Hewlett & Hibbert, 1967, Hewlett & Troedle, 1975; DeVries & Chow, 1978; Mosley, 1979; Pearce *et al.*, 1986; Sklash *et al.*, 1986; Tsukamoto & Ohta, 1988; Tanaka *et al.*, 1988) have observed the generation of runoff through the soil matrix "translatory flow" and "soil pipes" in mountainous forested hillslopes in humid regions and they confirm that these subsurface runoff mechanisms are capable of producing the observed high response of mountainous watersheds.

Many studies devoted to the problem of subsurface hillslope soil matrix drainage have made use of equation (1) which was first developed by Boussinesq (1877):

$$q = -Kh\left[\frac{\mathrm{d}h}{\mathrm{d}x}\cos\theta + \sin\theta\right] \tag{1}$$

where q is the flow rate per unit width of aquifer, K the hydraulic conductivity, h the thickness of the saturated zone measured perpendicular to the impermeable layer, and θ the slope angle of the impermeable layer. However, in most of these studies (Henderson & Wooding, 1964; Beven, 1981) the approach was further simplified by the kinematic wave approximation. Also, Sloan & Moore (1984) & Stagnitti *et al.* (1986) used this approximation to describe hillslope drainage as partly saturated flow. Because in the kinematic wave approximation the hydraulic gradient is assumed to be equal to $\sin\theta$ and the $\mathrm{d}h/\mathrm{d}x$ term in equation (1) is neglected, this approximation produces a zero flow for any horizontal aquifer, but this approximation is reasonable for steep mountainous hillslopes where the most important slope is the hill slope.

The dynamics of "pipe flow" are more complicated than soil matrix flow. However, Loukas & Quick (1993) showed that subsurface pipes can respond, under certain conditions, in a similar way to soil matrix flow. Loukas & Quick (1993) developed an equation for hillslope outflow from soil pipes by using kinematic wave dynamics which resembles the kinematic wave approximation of equation (1). Hathorn (1993) suggested that the flow rate per unit width through a soil containing soil pipes can be expressed as:

$$q = -K_{av} h \sin\theta \tag{2}$$

where K_{av} is the average saturated hydraulic conductivity of soil pipes and soil matrix.

From kinematic wave theory:

$$\frac{dh}{dt} = i_e \tag{3}$$

where i_e is the effective rainfall intensity and t is time. Integrating equation (3) and substituting into equation (2) gives:

$$q = -K_{av} i_e t S_H \tag{4}$$

where $\sin\theta$ has been substituted by S_H, the hillside slope.

Field experiments have indicated that the soil matrix hydraulic conductivity always decreases with depth in a given hillslope (DeVries & Chow, 1978; Beven, 1981) but in contrast the density of soil pipe networks, and thus their hydraulic conductivity, increases with depth (Tsukamoto & Ohta, 1988). As a result an average value of saturated hydraulic conductivity may be representative of the whole hillslope profile. The value of the average saturated hydraulic conductivity of soil pipes and soil matrix has to be measured in the field. Such a study in a forested watershed in Coastal British Columbia (Chamberlin, 1972) indicated that the downslope saturated hydraulic conductivity of soil containing soil pipes is about 350 mm h^{-1} which is more than double the 142 mm h^{-1} of the saturated hydraulic conductivity of the same soil without soil pipes (O'Loughlin, 1972). The values of K_{av} can vary significantly from soil to soil but in this study it would be assumed that K_{av} ranges around a value of 300 mm h^{-1}.

Equation (4) gives the outflow from an inclined hillslope and is valid when the duration of the storm, t_d, is less than the time when steady state conditions have been reached in the hillslope, t_s. When a steady state is reached, the flow profile will be given by:

$$q = i_e x = -K_{av} h_s S_H \tag{5}$$

where x is the distance from the beginning of the hillslope and h_s is the thickness of the saturated zone at the time when the steady state has been reached. At that time all the hillslope contributes to flow and so $x = L_H$, where L_H is the length of the hillslope. Also integrating equation (3) results in $h = i_e t$. Substituting into equation (5) and rearranging gives:

$$t_s = \frac{L_H}{K_{av} S_H} \tag{6}$$

which is the time when the steady state has been reached for the hillslope flow conditions. For any natural hillslope the value of t_s is always larger than the duration of any storm so equation (4) is always valid.

It should be mentioned that overland flow may occur on the hillslope when the saturation level, h, reaches the top of the surface of the hillslope. In steep forested hillslopes the soil depth, D, is at 1.5-2 m. According to equation (3), the time required for the saturation level to reach the surface of the hillslope is much longer than the duration of any storm. For example, if D = 1.5 m and i_e = 20 mm h^{-1}, then $t = D/i_e$ = 1.5/20 m/mm h^{-1} = 75 h which represents a storm with a total accumulation of 1500 mm which is very high for most areas of the world. Consequently, it is very unlikely that there will be free water on the surface of a hillslope and consequent surface runoff. This is in agreement with experiments in forested hillslopes where no overland flow has been observed (DeVries & Chow, 1978).

Stream channel phase

The watershed can be conceptualized as two planes draining into the main stem of the stream system. In the development of the equation for the lag time several assumptions have been made for the conditions in the stream channel, as follows.

It is assumed that the flow in the stream channel is kinematic. This is not an unrealistic assumption for steep mountainous watersheds where there is little storage in the stream and the flow is translated downstream with minimum attenuation. In addition, Manning's equation can be assumed to be valid for the stream flow:

$$Q = \frac{A R^{2/3} S_s^{1/2}}{n} \tag{7}$$

where Q is the stream flow, A is the wetted cross sectional area of the stream channel, R is the hydraulic radius, S_s is stream channel slope and n is the Manning roughness coefficient.

Also, it is assumed that the hydraulic radius of the stream channel can be expressed as a function of the wetted cross sectional area of the stream channel, A:

$$R = kA^{1/2} \tag{8}$$

where k is a shape factor.

This assumed relationship was explored further by assuming that the channels may have either a triangular or trapezoidal shape. The hydraulic radii were computed for a range of bank slopes and the shape factors are shown in Table 1. The values of bank slope of the stream channel should be determined by observations in the field.

Table 1 Values of channel shape factor (k) for triangular and trapezoidal cross sections

Bank slope (m)	Triangular cross section	Trapezoidal cross section $b_s = 1$	$b_s = 5$	$b_s = 10$	$b_s = 20$
0.40	0.294	0.335	0.371	0.357	0.317
0.60	0.332	0.357	0.374	0.357	0.315
0.80	0.349	0.363	0.371	0.352	0.312
1.00	0.354	0.363	0.364	0.345	0.306
1.25	0.349	0.355	0.352	0.335	0.299
1.50	0.340	0.343	0.339	0.323	0.291
1.75	0.328	0.330	0.326	0.312	0.283
2.00	0.316	0.318	0.313	0.301	0.275
2.5	0.294	0.294	0.291	0.281	0.260
2.75	0.283	0.284	0.281	0.272	0.253
3.00	0.274	0.274	0.271	0.264	0.247

b_s = base width (m)

Using continuity for the stream channel flow:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 2q \tag{9}$$

where Q is the stream flow, A is the wetted area of the channel and q is the lateral hillslope inflow per unit channel length which is defined in equation (4).

Application of the method of characteristics and substituting equations (7) and (8) result in the ordinary differentials:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}Q}{\mathrm{d}A} = \frac{4}{3}k^{2/3}A^{1/3}\frac{S_s^{1/2}}{n} \tag{10}$$

and

$$\frac{\mathrm{d}A}{\mathrm{d}t} = 2q \tag{11}$$

Substituting equation (4) into equation (11) and integrating results in:

$$A = K_{av}i_eS_Ht^2 \tag{12}$$

where t is measured from the beginning of inflow.

An assumption that is usually made in studies of the time response of watersheds is that both n and S_s are constant over the watershed, which is questionable because both n and S_s are likely to increase as one moves upstream. In this study both n and S_s are varied with the distance from the outlet of the watershed. Analysis of the main stream channel slope of 24 mountainous watersheds in Coastal British Columbia indicated that the longitudinal slope of the main stem of the stream increases with the distance from the outlet of the watershed and can be described by a second order polynomial:

$$S_s = a + bx + cx^2 \tag{13}$$

in which a, b, c are constants derived by curve fitting and x is the distance from the outlet of the watershed. In this study the stream slope of the main stem has been measured from 1:50 000 scale topographical maps but more detailed analysis can be done if field measurements are available and the watershed is accessible.

Forty-two measurements of Manning's n and water surface slope, S_w, in mountainous watersheds with mean stream slopes above 0.002 from across the USA were analysed. These data were taken from a recent publication (Hughes, 1993) which compiled data for natural streams published in two US Geological Survey reports (Barnes, 1967; Jarrett, 1985). Application of linear regression between Manning's roughness coefficient, n, and the water surface slope (Fig. 1) indicated that n can be written as:

$$n = 0.0326 + 1.3041 S_w \tag{14}$$

with $R^2 = 0.734$ and Standard Error of Estimate (SE) equal to 0.124.

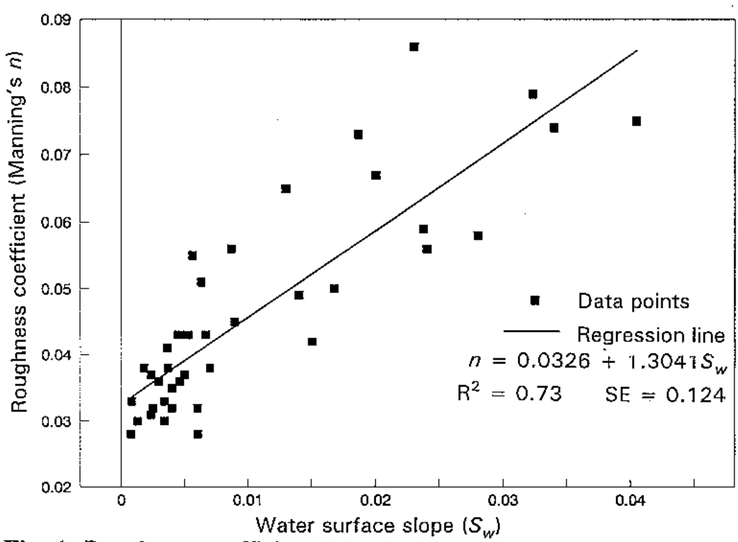

Fig. 1 Roughness coefficient (Manning's n) as a function of water surface slope (using data from Hughes, 1993).

The water surface slope, S_w, can be substituted by the stream slope S_s. This assumption can be quite weak for short irregular channel reaches; however it is acceptable when channel slopes are averaged over longer distances of the order of hundreds of metres (Hughes, 1993). Also, it is in agreement with the kinematic wave approximation assumed in this study for stream channel flow and the results of field research on mountain streams (Jarrett, 1987).

Substituting equation (13) into equation (14) results in:

$$n = p + qx + mx^2 \tag{15}$$

where $p = 0.0326 + 1.3041a$, $q = 1.3041b$ and $m = 1.3041c$.

Combining equations (10), (12), (13) and (15) gives:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{4}{3}k^{2/3}(K_{av}i_eS_H)^{1/3}t^{2/3}\frac{(a+bx+cx^2)^{1/2}}{p+qx+mx^2} \tag{16}$$

Rearranging equation (16) and integrating along the length of the main stem of the stream from 0 to L will require integration of the time from 0 to lag time, t_l:

$$\begin{aligned}\int_0^L \frac{p+qx+mx^2}{(a+bx+cx^2)^{1/2}}\mathrm{d}x &= \frac{4}{3}k^{2/3}(K_{av}i_eS_H)^{1/3}\int_0^{t_l} t^{2/3}\mathrm{d}t \\ &= \frac{4}{5}k^{2/3}(K_{av}i_eS_H)^{1/3}t_l^{5/3}\end{aligned} \tag{17}$$

The left hand side of equation (17) can be written as:

$$\begin{aligned}B &= \int_0^L \frac{p+qx+mx^2}{(a+bx+cx^2)^{1/2}}\mathrm{d}x = \int_0^L \frac{p\mathrm{d}x}{(a+bx+cx^2)^{1/2}} \\ &+ \int_0^L \frac{qx\mathrm{d}x}{(a+bx+cx^2)^{1/2}} + \int_0^L \frac{mx^2\mathrm{d}x}{(a+bx+cx^2)^{1/2}}\end{aligned} \tag{18}$$

$$\Rightarrow B = B_1 + B_2 + B_3$$

Integration of equation (18) yields:

$$B_1 = \frac{p}{a^{1/2}}B_k \tag{19}$$

where $B_k = \ln[2a^{1/2}(a + bL + cL^2) + 2aL + b] - \ln(2a^{1/2}c^{1/2} + b)$.

$$B_2 = \frac{q(a+bL+cL^2)}{a} - \frac{qb}{2a^{3/2}}B_k - \frac{qc^{1/2}}{a} \tag{20}$$

$$B_3 = m\left[\frac{2aL-3b}{4a^2}(a+bL+cL^2) + \frac{3bc^{1/2}}{4a^2} + \frac{3b^2-4ac}{8a^{5/2}}B_k\right] \tag{21}$$

B_1, B_2, B_3 can be found from the known parameters of a given watershed. Substituting back into equation (17) and rearranging:

$$t_l = \left[\frac{5B}{4k^{2/3}(K_{av}i_eS_H)^{1/3}}\right]^{3/5} \tag{22}$$

where $B = B_1 + B_2 + B_3$.

Taking length in metres, rain intensity in millimetres per hour, and time in minutes, equation (22) takes the form:

$$t_l = 4.32 \frac{B^{0.6}}{k^{0.4}(K_{av} i_e S_H)^{0.2}} \tag{23}$$

Equation (23) computes the lag time of watersheds if the topographic characteristics, rainfall intensity and the average saturated hydraulic conductivity are known. It links the physical characteristics of a watershed to its time response through an analytical mathematical procedure. The ability of equation (23) to predict the lag time of two experimental watersheds along with other empirical equations will be tested next.

APPLICATION OF THE DERIVED EQUATION AND COMPARISON WITH OTHER EMPIRICAL EQUATIONS

Equation (23) derived for the estimation of the lag time has been applied to two experimental watersheds to test its validity. Also in this section, other commonly used empirical and analytical equations have been used to compute the lag time of the two watersheds and the results were compared with the observed lag time and with the results predicted by the proposed equation. This comparative study will show that inaccurate estimates of the time response can result from using equations developed for watersheds different from the forested mountainous watersheds of this study.

Study watersheds

The two experimental watersheds were Carnation Creek and Jamieson Creek, located in Coastal British Columbia. The Carnation Creek watershed is on the west coast of Vancouver Island. The basin area of 9.5 km^2 features rugged terrain from sea level to 880 m a.m.s.l. with steep slopes of 40% to over 80% and a relatively wide valley bottom. Slope soils are coarse colluvial materials of gravelly loam to loamy sand texture with a moderately thick organic layer and are underlain by bedrock of volcanic origin (Hetherington, 1982).

The second study watershed, Jamieson Creek, is located about 30 km north of Vancouver. The basin has an area of 3 km^2 and its elevation ranges from 305 to 1310 m a.m.s.l.. Jamieson Creek is characterized by steep slopes with an average hillslope gradient of 48%. The soils of the watershed are shallow and very permeable coarse-textured sand and gravelly, sandy loam underlain by bedrock (Cheng, 1975).

The climate of the coastal region of British Columbia is influenced by the adjacent Pacific Ocean and features mild, wet winters, characterized by frequent frontal storms, and mild summers. Annual precipitation is mostly rain and ranges from about 2100 mm to over 4800 mm, over 75% falling in October to March. Snow occurs at high elevations but it is wet and ripe throughout the winter period.

The data from Carnation Creek consist of hourly streamflow and hourly precipitation from one recording station. In addition, there are six high elevation storage gauges that have been used to calculate the ratios of the long term annual precipitation to that of the recording station. In Jamieson Creek there are five recording precipitation stations which measure hourly precipitation and a streamflow gauge at the mouth of the watershed which records hourly flows.

Watershed modelling of Jamieson Creek runoff response (Loukas & Quick, 1993) indicated that, on average, 1.4 mm h^{-1} are diverted to the slow or groundwater runoff. Also, that study indicated that the Hewlett & Hibbert (1967) hydrograph separation method is valid for that watershed. According to the Hewlett & Hibbert method the hydrograph can be separated into its fast and slow runoff components by drawing a line upward from the point of initial hydrograph rise at a slope of 0.55×10^{-3} m^3 s^{-1} km^{-2} h^{-1}. For the Jamieson Creek watershed the rainfall abstractions were taken as the abstractions computed in the previous rainfall-runoff study (Loukas & Quick, 1993) whereas for the Carnation Creek watershed they were assumed to be, on average, equal to 1.4 mm h^{-1}. The direct runoff hydrograph for both watersheds was calculated by using the Hewlett & Hibbert method. The rainfall-runoff events analysed were single peak hydrograph events. Sixteen events were analysed for the Carnation Creek watershed and seven events for the Jamieson Creek watershed.

The slope of the main stream of the two study watersheds was plotted against the distance from the outlet of the watershed and second order

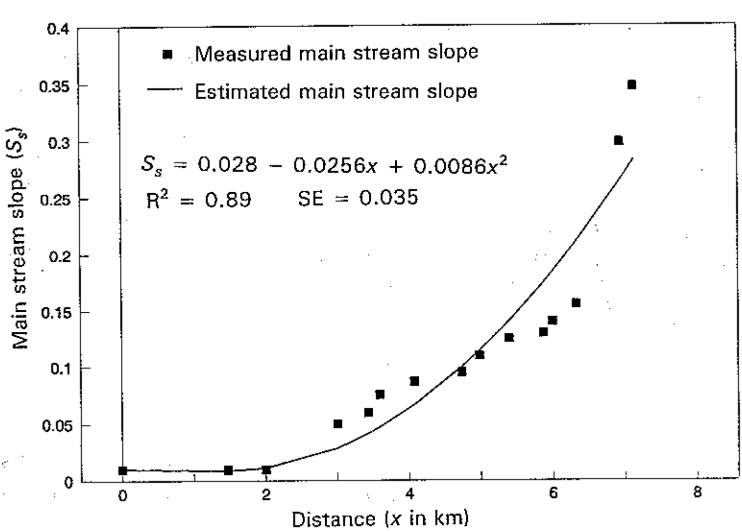

Fig. 2 The main stream slope as a function of the distance from the outlet of the Carnation Creek watershed.

polynomials were fitted (Figs 2 and 3) as discussed earlier and presented in equation (23). The coefficients a, b, c of equation (23) are shown in Table 2 along with the values of other measured or estimated parameters for the two watersheds.

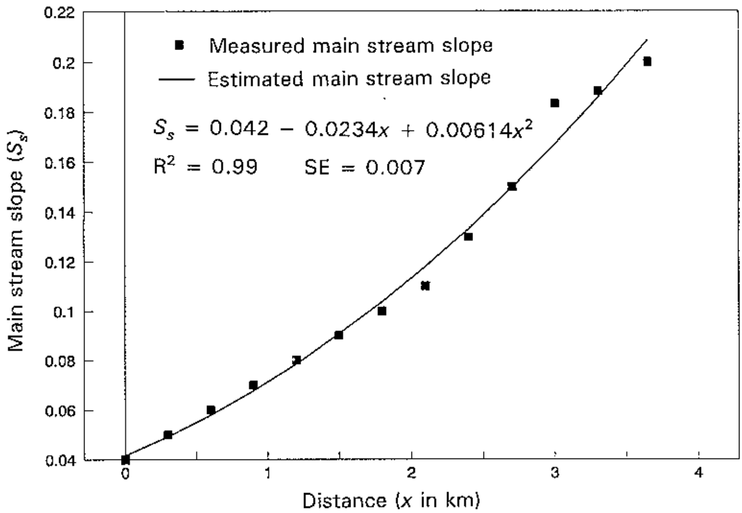

Fig. 3 The main stream slope as a function of the distance from the outlet of the Jamieson Creek watershed.

Table 2 Characteristic parameters of the two study watersheds

Parameters	Carnation Creek	Jamieson Creek
Length of stream, L (m)	7470	3645
Mean stream slope, S_s	0.08	0.16
Coefficient a in equation (13)	0.028	0.042
Coefficient b in equation (13)	-2.5×10^{-5}	-2.34×10^{-5}
Coefficient c in equation (13)	86.0×10^{-10}	61.4×10^{-10}
Manning's n	0.137	0.24
Watershed shape factor, f ($m^{-1/5}$)	0.47	0.47
Channel shape factor, k	0.35	0.35
Average soil hydraulic conductivity, K_{av} (mm h^{-1})	300	300
Average hillside slope, S_H	0.40	0.48
B in equation (23)	2684	841.6

Methods for estimating lag time

Seven methods were selected for the computation of lag time of the two study watersheds and for the comparison with the results of the proposed equation

(23). Five of the methods calculate the lag time of the watershed whereas two calculate the time of concentration. For these latter two equations the lag time was assumed to be 60% of the time of concentration. Also, five equations compute the lag time as a constant parameter of the watershed and they link the lag time to only the physiographic parameters of the watershed, such as the main stream length, L, and the mean stream slope, S_s, whereas the other two methods link the lag time to rainfall intensity as well. In the next paragraphs a brief presentation of the selected methods is given.

Kiprich equation Kiprich (1940) presented one of the first published works on the time of concentration of a watershed. Using data from six small basins in Tennessee, USA, which ranged in area from 0.0051 to 0.433 km^2, he developed a graphic correlation of $L/\sqrt{S_s}$ with time of concentration. Kiprich suggested that the curves would be applicable to the average small agricultural area ranging in size from 0.003 to 0.5 km^2. Based on these and other data, Rowe & Thomas (1942) obtained the following equation:

$$t_c = 0.000325 \left[\frac{L}{\sqrt{S_s}} \right]^{0.77} \tag{24}$$

where t_c is in h, L in m and S_s is dimensionless. This equation, which is often referred to as Kiprich's equation, is widely quoted and has received widespread use both in simulation models and in design manuals for small drainage basins (Watt & Chow, 1985).

Chow equation As part of a project to develop a method for estimating design floods in Illinois, USA, Chow (1962) analysed data from 20 basins in Illinois, Ohio, Missouri, Wisconsin, Indiana, Iowa and Nebraska, with areas ranging from 0.012 to 18.5 km^2. He developed the following prediction equation:

$$t_l = 0.00116 \left[\frac{L}{\sqrt{S_s}} \right]^{0.64} \tag{25}$$

where t_l is in h, L in m, and S_s is dimensionless. This equation has not been as widely quoted as Kiprich's equation even though it is based on data from basins with a wide range of area and in various geographical regions.

Natural Environment Research Council (NERC) equation In the Flood Studies Report (Natural Environment Research Council, 1975), the following prediction equation is given for application to ungauged basins in the United Kingdom:

$$t_l = 2.8 \left[\frac{L}{\sqrt{S_s}} \right]^{0.47} \tag{26}$$

where t_l is in h, L in km, and S_s in m km^{-1}.

Modified Snyder equation In a study of basins in the Appalachian Mountain region, Snyder (1938) found that the basin lag is a function of basin size and shape. The equation that Snyder proposed was modified by the US Army Corps of Engineers (Linsley *et al.*, 1982) and took the form:

$$t_l = C_b \left[\frac{L L_c}{\sqrt{S_s}} \right]^{0.38} \tag{27}$$

where t_l is in h, L and L_c (the stream distance from outlet to a point opposite the basin centroid) in km, S_s is dimensionless and C_b is a coefficient which depends on the average resistance to flow through the drainage network. The US Army Corps of Engineers (1990) proposed various values for C_b. A value of C_b equal to 0.42 was assumed in this study. The modified Snyder equation has been used widely in watershed modelling and design considerations.

Watt & Chow equation Watt & Chow (1985) developed an equation for the lag time of basins by using data from 44 basins across the USA and Canada in the form:

$$t_l = 0.000326 \left[\frac{L}{\sqrt{S_s}} \right]^{0.79} \tag{28}$$

where t_l is in h, L in m, and S_s is dimensionless. The equation was developed with data from basins that ranged from 0.01 to 5840 km^2 in area and with slope of main stream ranging between 0.00121 and 0.0978.

Aron *et al.* equation Aron *et al.* (1991) developed an equation for the lag time by using kinematic wave dynamics and certain assumptions. They identified the overland flow as the main runoff generation mechanism and used the rational equation to develop the equation:

$$t_l = 0.93 \frac{k^{5/12} n^{3/4} L^{7/12}}{c^{1/2} i_e^{1/4} S_s^{3/8}} \tag{29}$$

where t_l is in min, L in m, S_s is dimensionless, i_e is in mm h^{-1}, the watershed shape factor $k = L/A^{3/5}$ is in m$^{-1/5}$, where A is in m^2, and the channel shape factor, c, depends on the depth-width ratio.

Papadakis & Kazan equation Papadakis & Kazan (1987) analysed data from 84 rural watersheds with areas less than 5 km^2, and experimental data from the US Army Corps of Engineers, Colorado State University and the University of Illinois, Urbana-Champaign. In total 375 data points were analysed and a regression equation fitted to the data for the time of concentration took the form:

$$t_c = 0.66 L^{0.5} n^{0.52} S_s^{-0.31} i_e^{-0.38} \tag{30}$$

where t_c is in min, L in feet, S_s is dimensionless, and i_e is in in h^{-1}.

Application of the proposed equation and the other methods

The above equations were applied along with the proposed equation (23) to the two study watersheds. Sixteen single peak events were analysed and used from the Carnation Creek watershed whereas seven events were used from the Jamieson Creek database. The storm events used had a return period of less than ten years but they are considered to be representative of the flow regime of the two watersheds.

Various statistical criteria were used to measure the bias and the precision of the various methods. For the proposed equation (23) and the Aron *et al.* and Papadakis & Kazan methods, the statistical criterion for bias was the standardized bias:

$$B_s = \frac{1}{m}\sum_{i=1}^{m}\frac{\hat{t}_l - t_l}{t_l} \tag{31}$$

in which m is the number of events, and $\hat{t}_l$ and are the computed and measured values of lag time.

The accuracy of a method is also an important criterion. The first five methods, (Kiprich, Chow, NERC, Modified Snyder and Watt & Chow) assume that the time response of a watershed is constant and the results of these methods were compared with the average observed lag time. To assess the accuracy of the proposed equation (23) and the Aron *et al.* and Papadakis & Kazan methods, the mean error and the standard deviation of errors were computed. Also, the standard error was used in the comparison defined as:

$$SE = \left[\frac{1}{m}\sum_{i=1}^{m}\left(\frac{\hat{t}_l - t_l}{t_l}\right)^2\right]^{1/2} \tag{32}$$

The values of the lag time computed with the proposed equation (23) and the other methods are compared with the observed lag time in Tables 3 and 4. These results indicate that the proposed equation (23) computes the lag time of the two study watersheds with improved accuracy and precision over the other methods. The proposed equation has the smallest values of standardized bias, close to zero, which indicates that the proposed method does not severely overestimate or underestimate the observed lag time. In addition, the proposed method approximates the observed mean lag time within −0.59 h for the Carnation Creek watershed and 0.09 h for the Jamieson Creek watershed. The other statistics support the conclusion that the proposed equation (23) gives better results than the other methods.

The results in Tables 3 and 4 clearly indicate that the other methods severely under-predict the lag time of the two study watersheds by as much as 90%. Only the Papadakis & Kazan equation reasonably approximates the observed lag time of the Jamieson Creek watershed, but on the other hand it underestimates the lag time of the Carnation Creek watershed by about 57%.

Table 3 Goodness-of-fit statistics for the test of lag time formulae: Carnation Creek watershed

Method	Mean lag time (h)	Mean error (h)	Standard deviation of errors (h)	Standardized bias (B_s)	Standard error
Observed	4.64	-	-	-	-
Proposed equation (23)	4.05	−0.59	1.48	−0.02	0.35
Aron *et al.* equation (29)	1.63	−3.01	1.49	−0.60	0.63
Papadakis & Kazan equation (30)	2.00	−2.64	-	−0.52	0.55
Kiprich equation (24)	0.49	−4.15	-	-	-
Chow equation (25)	0.78	−3.86	-	-	-
NERC equation (26)	2.57	−2.07	-	-	-
Modified Snyder equation (27)	2.40	−2.24	-	-	-
Watt & Chow equation (28)	1.01	−3.63	-	-	-

Table 4 Goodness-of-fit statistics for the test of lat time formulae: Jamieson Creek watershed

Method	Mean lag time (h)	Mean error (h)	Standard deviation of errors (h)	Standardized bias (B_s)	Standard error
Observed	1.99	-	-	-	-
Proposed equation (23)	2.08	0.09	0.35	0.08	0.16
Aron *et al.* equation (29)	1.36	−0.63	0.38	−0.30	0.30
Papadakis & Kazan equation (30)	1.71	−0.28	0.28	−0.13	0.15
Kiprich equation (24)	0.22	−1.77	-	-	-
Chow equation (25)	0.40	−1.59	-	-	-
NERC equation (26)	1.56	−0.43	-	-	-
Modified Snyder equation (27)	1.22	−0.76	-	-	-
Watt & Chow equation (28)	0.44	−1.55	-	-	-

Probably this discrepancy occurs because this equation was developed for small rural watersheds with areas less than 5 km^2. The Aron *et al.* equation, which is analytical, does not perform well because it assumes that overland flow is the main runoff generation mechanism which is not valid for forested mountainous watersheds. The rest of the empirical equations severely underestimate the observed lag time.

CONCLUSIONS

An equation has been derived to compute the lag time of forested mountainous watersheds. In the development of this equation, the flow in a watershed is separated into two phases, the land phase which represents the generation of flow in the hillslopes and the stream channel phase. In the land phase the runoff is generated through subsurface pathways either soil matrix flow or flow through macropores or soil pipes. Using observations and findings reported in field experimental studies and assuming kinematic conditions an equation for the outflow from the base of a hillslope has been presented. This outflow is used as inflow to the stream. For the solution of the flow equations in the stream channel, kinematic conditions have been assumed. In addition it was assumed that Manning's n and main stream slope vary with distance from the outlet of the watershed. Data from the USA and Coastal British Columbia were used to validate these assumptions. Finally, solving the resulting partial differentials, the proposed equation for the estimation of lag time was derived.

Application of this equation to two experimental study watersheds in Coastal British Columbia indicated that the method is reliable and approximates well the observed lag time of the watersheds. In this study, it was also shown that other empirical and analytical equations, usually developed for agricultural watersheds, severely underestimated the lag time when applied to forested mountainous watersheds. This inaccuracy of lag time can lead to severe overestimation of the peak flow in design flood computations.

The main advantage of the proposed equation is that it has been derived by using observations from field experiments and by applying simplified mathematics whereas the empirical equations are, usually, the product of regression analyses between the observed lag time and topographic and rainfall parameters. Application of the proposed method minimizes the errors in lag time computation and so results in better, more reliable prediction of the design flood. Also, the proposed equation has been derived for forested mountainous watersheds and because of its analytical derivation it may be applicable to broad geographical and climatic regions.

However, certain precautions should be taken when applying the proposed equation (23). Firstly, its application should be restricted to steep forested mountainous watersheds where the slopes of stream and hillsides dominate the equations of motion. The authors' extensive research on mountain rivers has shown that the flow in these conditions is translated with only small to minimal attenuation, so kinematic wave theory is valid. This observation has been supported by other researchers (Jarrett, 1987; Kellerhals, 1970). Further, the proposed equation should be applied to areas where storm flow is generated through subsurface mechanisms because this is the main assumption in the derivation of the equation. Secondly, the testing of the proposed equation was made with good quality, but limited data, and needs to be tested with additional data from other geographical regions. Thirdly, the storms used for the validation of the equation had a return period of less than ten years. Hence, caution should be used when the equation is applied to more extreme storms and its validity for such events should be examined when data become available.

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