Height-Age Curves for Interior Spruce in British Columbia

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Executive Summary

Height-age and site-index curves were developed from stem-analysis data of three or four trees in 109 plots. The plots were located throughout B.C., but were mostly in the 15- and 20-m site-index class in the Sub-Boreal Spruce Biogeoclimatic zone. The data used in formulation of the new height-age and site-index curves ranged in site index from about 6-26 m up to 145 years of age. The height-age model chosen to best describe the data were selected from several variations of the conditioned logistic function as having the best pattern of residuals and extrapolations to 250 years of age. Several nonlinear models were also tested for estimating site index as a function of height and age.

The age-adjusted conditioned logistic function was selected for describing the height-growth patterns of free-growing interior spruce. This model best the best before and after index age (50 years breast-height age) and gave slightly more conservative estimates at older ages than the other formulations. The formulated height-age curves were similar to the currently used interior spruce curves (Goudie 1984) between about 10 and 80 years breast-height age. However, the new curves show slightly greater heights below 10 years and after 80 years. Residual analyses showed that Goudie's curves under-estimate the height of the stem-analysis trees for the given site index, and thus the new curves should provide improved estimates of the height growth for interior spruce.

An equation was developed to estimate directly site index, however, the accuracy and precision of the model was relatively poor. Iterative methods used to estimate site index using the height-age curves gave better estimates than the site-index equation. Thus it is recommended that site index of interior spruce is estimated using the new height-age curves. Estimates of site index using this method are lower by about 0.5-1.0 m at 100 years breast-height age and by about 2.0-2.5 m at 250 years than when using Goudie's curves and the same technique (the method currently used in B.C.). This reflects the slight differences in the height-age curves and the improved estimates at older ages from the new-curves.

Table of Contents

List of Tables

List of Figures

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INTRODUCTION

The B.C. Ministry of Forests has undertaken an aggressive program of developing new height-age (site-index) curves over the past several years. The intent is to develop new curves using stem-analysis data collected from throughout the geographic and ecological range for each commercial species in the Province. The currently used height-age curves for interior spruce were developed by Goudie (1984) using stem-analysis of individual trees and formulated with a conditioned logistic function. Of the 157 trees used in Goudie's analysis, 125 (80%) were from Alberta and the remaining 32 (20%) were from B.C. This project to develop new height-age curves for interior spruce was initiated by Inventory Branch, Research Branch, and the Forest Productivity Councils to update Goudie's curves by collecting data from B.C. and to formulate new curves using the local data. This report describes the data and the procedures used in the development of these curves.

METHODS

Plot Data

Three data sources were used in the development of the new height-age and site-index curves. The collection of all data was funded by the Inventory Branch of the B.C. Ministry of Forests. These were data from: i) Industrial Forestry Service (IFS); ii) Sylva Management (Sylva); and iii) University of British Columbia (UBC). The data were collected throughout the range of the species; however, most plots were in the 15 and 20 m site-index class in the SBS biogeoclimatic zone (Table 1).

Table 1. Number of plots by Biogeoclimatic zone, 5-m site-index class, and data source.

BEC	5-m SI Class			10-m SI Class			15-m SI Class			20-m SI Class			25-m SI Class			
Zone	IFS	Sylva	UBC	IFS	Sylva	UBC	IFS	Sylva	UBC	IFS	Sylva	UBC	IFS	Sylva	UBC	Total
ESSF							3			4						7
ICH										1			3			4
IDF							3						1			4
MS							3			4						10
SBS			2		1	2		5	22	1	9	34	1		7	84
Total			2		1	2	9	5	22	10	9	34	5		7	109

IFS Data

These data were collected in 1989 under contract to Inventory Branch (IFS Contract No. 891090). The intent of the contractor was to collect the 31 interior spruce plots from across the range of site in the Nelson and Kamloops Forest Regions. The sample plots were 20x20 m where four dominant trees were felled for stem-analysis. Each site tree was sectioned at stump height (0.3 m), breast height (1.3 m), and at each decile point above breast height. Thus 11 disks were taken to reconstruct the height growth of each tree.

Sylva Data
Sylva Management were contracted by Inventory Branch to collect stem-analysis data from interior spruce in 1988/89. The 20 sample plots were taken in the SBS zone from Williams Lake north to Mackenzie and from Horsefly west to Bednesti. Plot size and sampling procedures were the same as for the IFS data.

UBC Data
These data were collected by Gaofeng Wang in the summer of 1990 for his Ph.D. dissertation at UBC with Dr. Karl Klinka. These plots were collected mainly in the southern portion of the Prince George Forest Region with some plots in the Burns Lake area (Prince Rupert Region) and the Quesnel area (Cariboo Forest Region). Plot size was also 20x20 m and site tree selection was the same as for the IFS and Sylva data sources. However, the stem-analysis was more intensive and disks were taken every one metre of height. Many of these plots were less than 50 years breast-height age and could not be used in the analysis.

Plot Height-Age Curves

Height-age curves were developed for the site trees in each plot based on total age and breast-height age. The height where the stem-analysis disks were cut from the tree bole was corrected for stem-analysis bias that occurs from not cutting the tree exactly where height growth ceases in that year (Carmean 1972). Tree total age was estimated by adding four years to the age of the stump height disk (0.3 m). The number of years to reach breast height (1.3 m) was computed as the difference between the age counted from the breast height disk and the estimated total age of the tree.

Total-age height-growth curves were then plotted to identify trees with irregular growth patterns. Trees having erratic height-age patterns and those indicating suppression were eliminated from the data (Table 2). All plots had at least three trees showing undamaged and unsuppressed height growth.

Table 2. Number of plots eliminated as too young or having irregular height-age patterns.

Data Source	Original No. of Plots	Eliminated as less than 50 years (BH-age)	Eliminated as having irregular ht-age patterns	No. of Plots used in the Analysis
IFS	31	none	4	27 (25%)
Sylva	20	none	5	15 (14%)
UBC	102	26	9	67 (61%)
Total	153	26	18	109 (100%)

The remaining trees and plots that were considered to reflect undamaged and unsuppressed height growth were adjusted to breast-height age. These curves were used to determine the height of each tree at 5-year intervals. The plot average height-age curve was constructed as the average of the heights at these 5-year intervals for the three or four site trees in each plot. The average curves was based on all trees in a plot, thus the age was limited by the youngest tree in the plot. Site index for the plot was computed as the average height of the site trees at breast-height age 50 years. Thus plots where all trees were not at least 50 years breast-height age were not included in the analysis. Average site index for the plots was 18.4 and ranged from 6.1-25.9 m.

Formulated Height-Age Curves

Three conditioned logistic functions were tested for describing the height-age patterns: the age-adjusted model:

[1] $$H = 1.3 + (SI-1.3)\left[\frac{1+e^{(b_1 + b_2\ln(A_{SI}+b_4) + b_3\ln(SI-1.3))}}{1+e^{(b_1 + b_2\ln(A+b_4) + b_3\ln(SI-1.3))}}\right]$$

the fixed-intercept model (the standard conditioned logistic model):

[2] $$H = 1.3 + (SI-1.3)\left[\frac{1+e^{(b_1 + b_2\ln A_{SI} + b_3\ln(SI-1.3))}}{1+e^{(b_1 + b_2\ln A + b_3\ln(SI-1.3))}}\right]$$

and the floating-intercept model:

[3] $$H = (a_0+a_1SI) + (SI-(a_0+a_1SI))\left[\frac{1+e^{(b_1 + b_2\ln A_{SI} + b_3\ln(SI-(a_0+a_1SI)))}}{1+e^{(b_1 + b_2\ln A + b_3\ln(SI-(a_0+a_1SI)))}}\right]$$

where H is height (m); a_i and b_i are regression coefficients; SI is the site index (dominant height (m) at 50 years breast-height age), A is breast-height age (yrs), and A_{SI} is the reference age of the site index (50 years breast-height age).

The height-age models were fit to the plot-average curve data (with observations of height at each 5-year interval) up to 145 years of breast-height age (Figure 1). The few data older than 145 years were not included because they were influential without necessarily being representative of overall height-growth patterns at these ages. The data also included heights at breast-height age zero. This is necessary for fitting the age-adjusted and floating-intercept model, and is also useful for showing the bias of the fixed-intercept model.

The model for describing the height-age patterns was selected as having the least bias and best patterns of residual variation, and also that provided reasonable extrapolations of height to age 250 years. These final height-age curves were then compared to the curves currently used in B.C. (Goudie 1984).

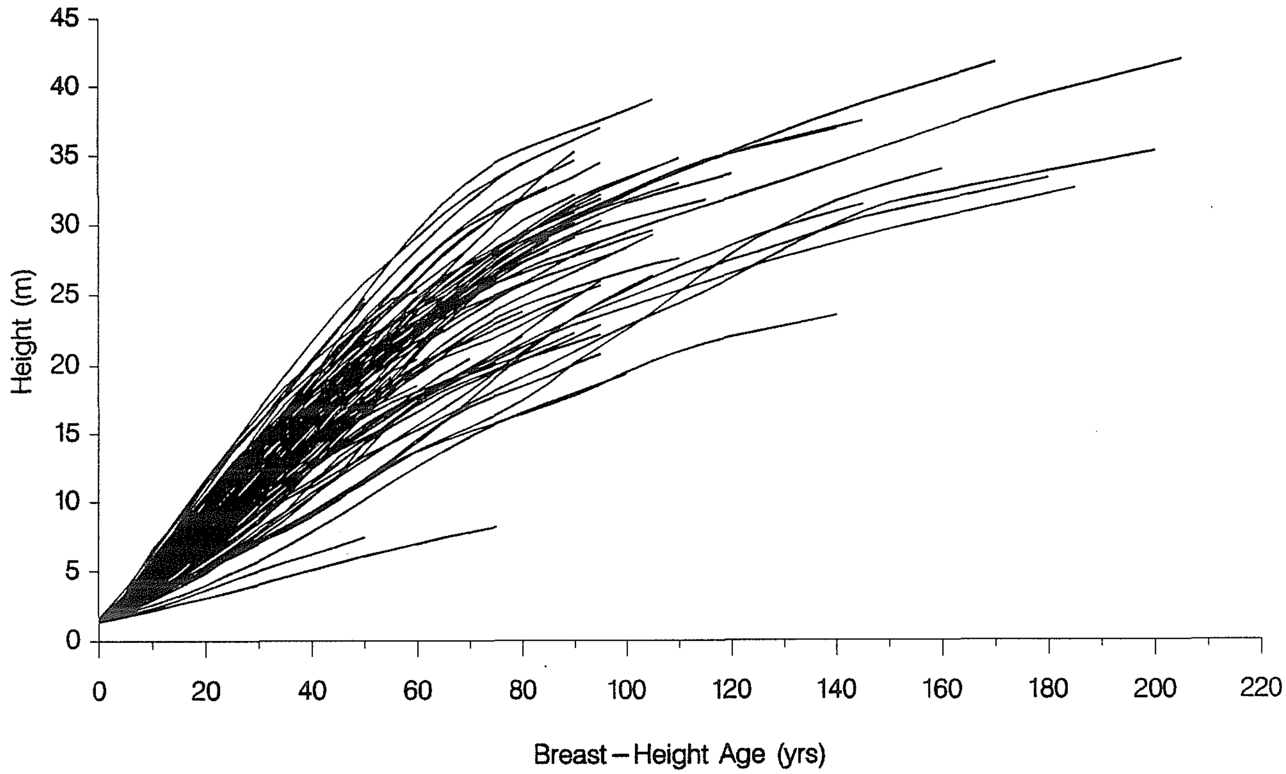

Figure 1. Breast-height age curves for the 109 interior spruce stem-analysis plots.

Formulated Site-Index Curves

A separate model was fit to the data describing site index as a function of height and breast-height age. The intent was to provide a model that could predict site index directly and thus give better estimates compared to indirectly estimating site index using iterative methods with the height-age curves.

Several models were tested for estimating site index including the conditioned logistic, Payendeh's (1974) modification of the Chapman Richards function, and various modifications of these models. The models were tested using data between 15 and 145 years of breast-height age. The height-age data at 5 and 10 years breast-height age were not included because they were very poorly related to site index and did not contribute to estimation of the parameters. The final site index model was selected as showing the lowest bias and the best patterns of residual variation.

RESULTS AND DISCUSSION

Height-Age Curves

Formulation

The model selected as providing the best description of the height-growth patterns of interior spruce was the age-adjusted conditioned logistic function:

$$[4] \qquad H = 1.3 + (SI - 1.3) \left[\frac{1 + e^{(10.464044 \; - \; 1.595737 \ln(50 + 3.155802) \; - \; 1.282344 \ln(SI - 1.3))}}{1 + e^{(10.464044 \; - \; 1.595737 \ln(A + 3.155802) \; - \; 1.282344 \ln(SI - 1.3))}} \right]$$

where *H* is dominant height (m); *SI* is the site index (dominant height (m) at 50 years breast-height age), and *A* is breast-height age (yrs). The formulated curves are shown in Appendix I and predicted heights are given in Appendix II.

The age-adjusted model was very similar to the fixed-intercept and the floating-intercept model in the statistical fit (Table 3) and in extrapolated heights (Figures 2 and 3). However, the age-adjusted model showed the lowest bias, the best over-all pattern of residuals over age (Figures 4 and 5) and site index (Figure 6), and the most realistic predictions at early ages (including breast-height age zero) (Figure 7).

Table 3. Regression coefficients and root mean squared error (RMSE) of the three height-age models.

Statistic/ Parameter	Height-Age Model		
	Age-Adjusted Model (Equation 1)	Fixed-Intercept Model (Equation 2)	Floating-Intercept Model (Equation 3)
RMSE	0.978	0.988	0.981
b_1	10.464044	10.020978	10.173500
b_2	-1.595737	-1.399189	-1.468994
b_3	-1.282344	-1.383247	-1.366477
b_4	3.155802		
a_0			1.687077
a_1			0.000894

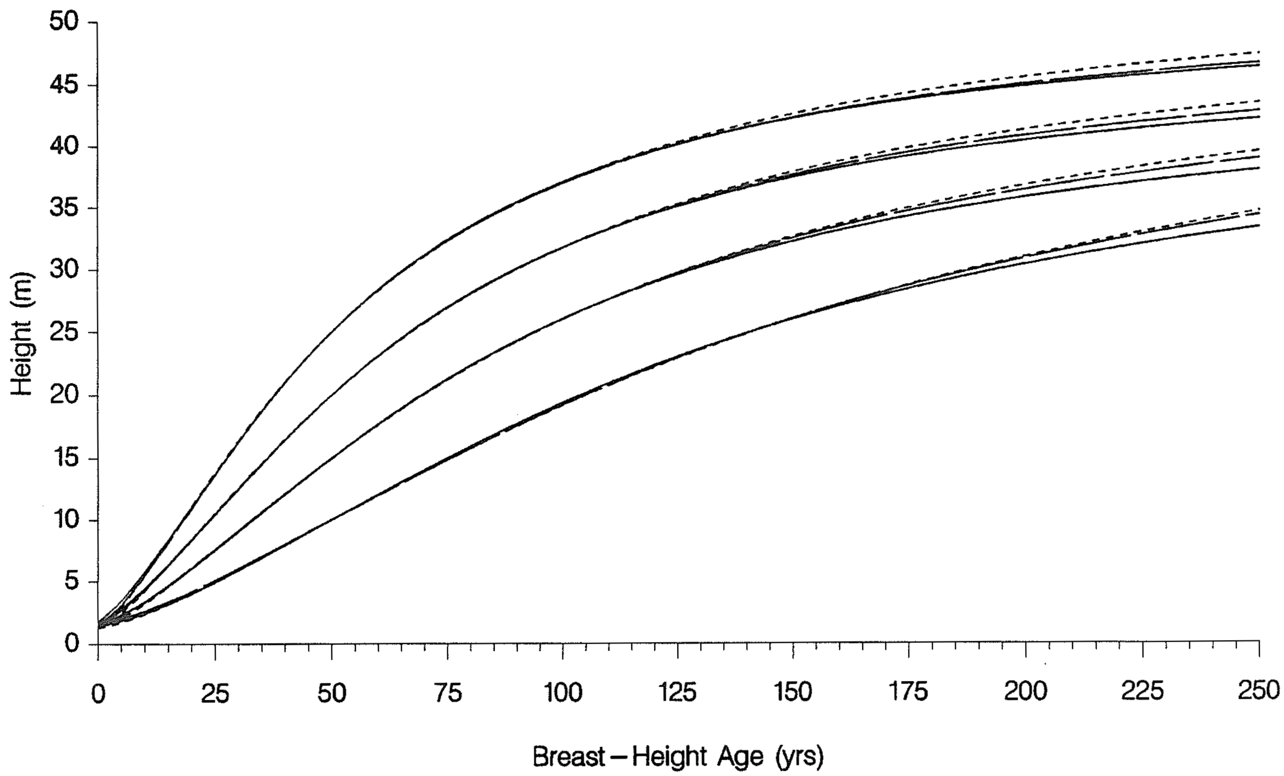

Figure 2. Formulated height-age curves for the age-adjusted model (solid line), the floating-intercept model (long-dashed line), and the fixed-intercept model (short-dashed line).

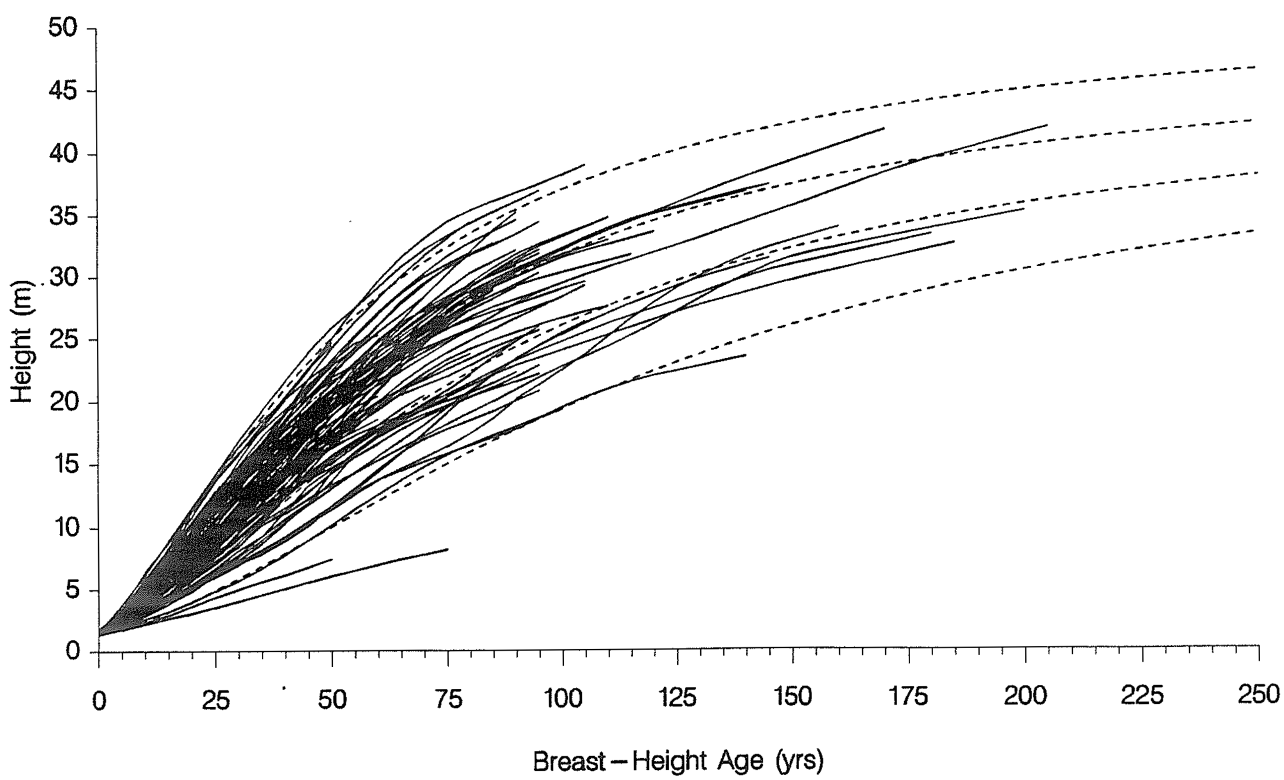

Figure 3. Height-age curves for the 109 interior spruce stem-analysis plots and the formulated curves with the age-adjusted model.

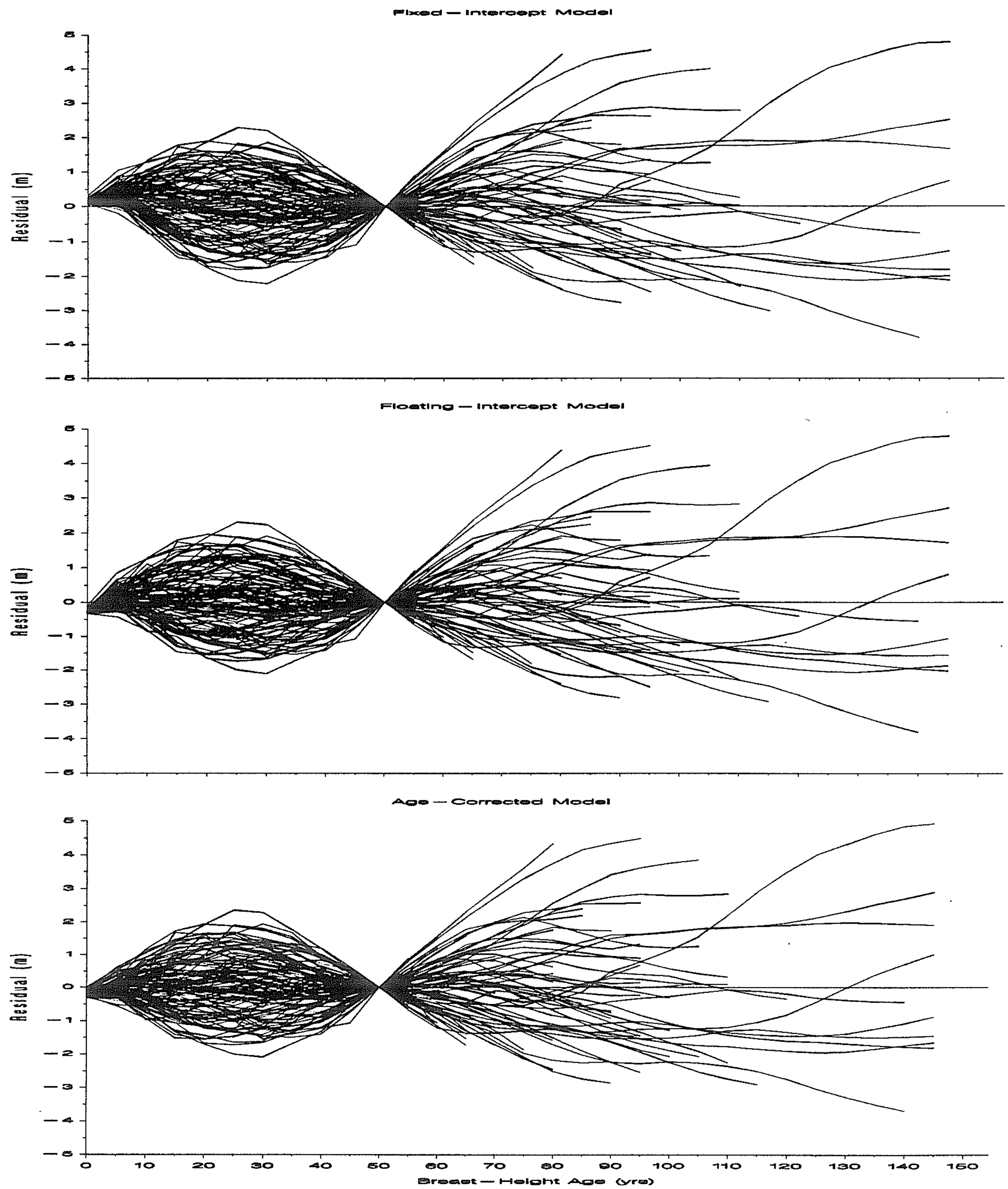

Figure 4. Spider plots showing residuals of height for the individual plots from the three models. Positive residuals indicate under-prediction of the model.

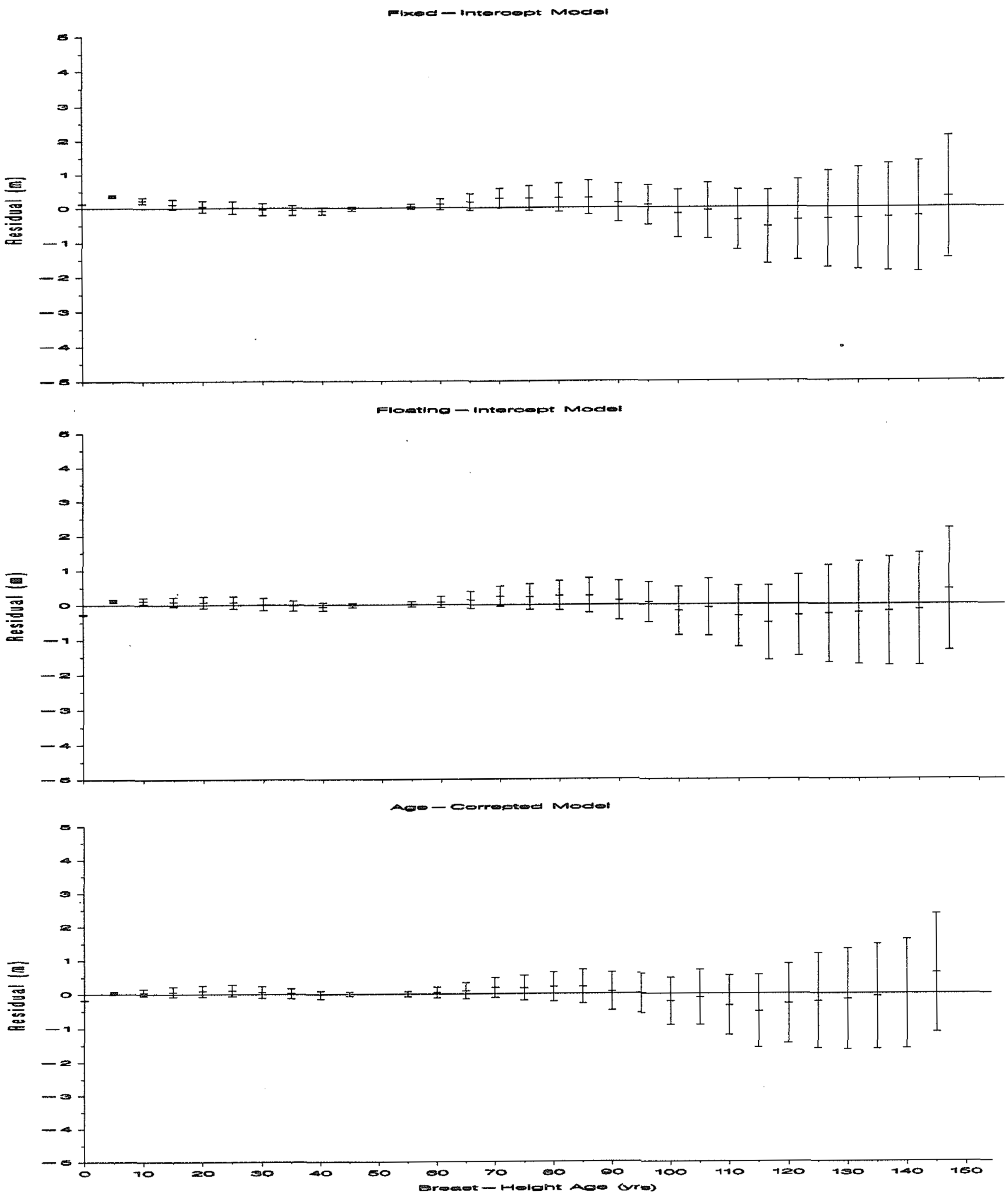

Figure 5. Mean and two standard errors of height residuals for the three models. Positive residuals indicate under-prediction of the model.

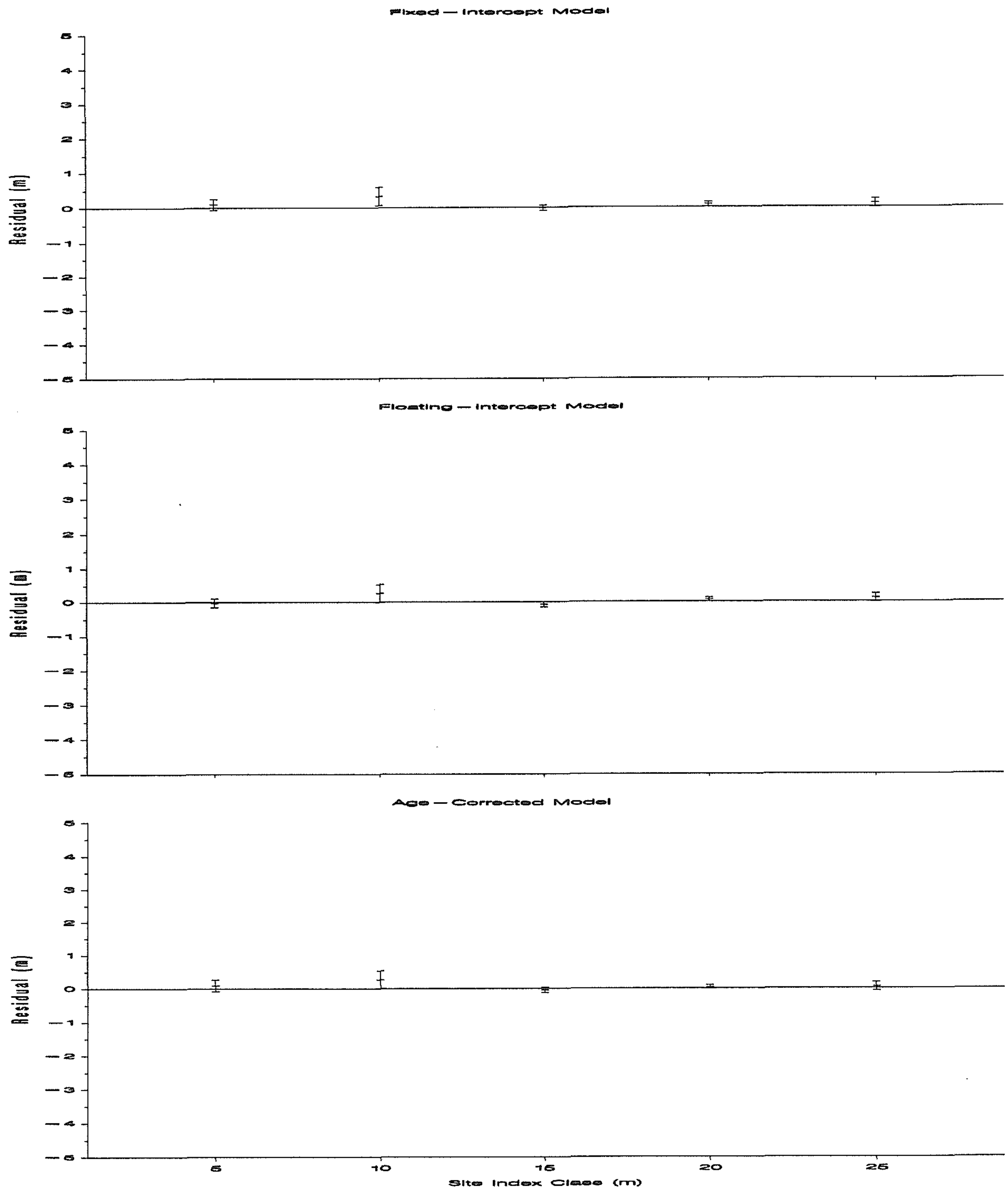

Figure 6. Mean and two standard errors of height residuals by 5-m site index class for the three models. Positive residuals indicate under-prediction of the model.

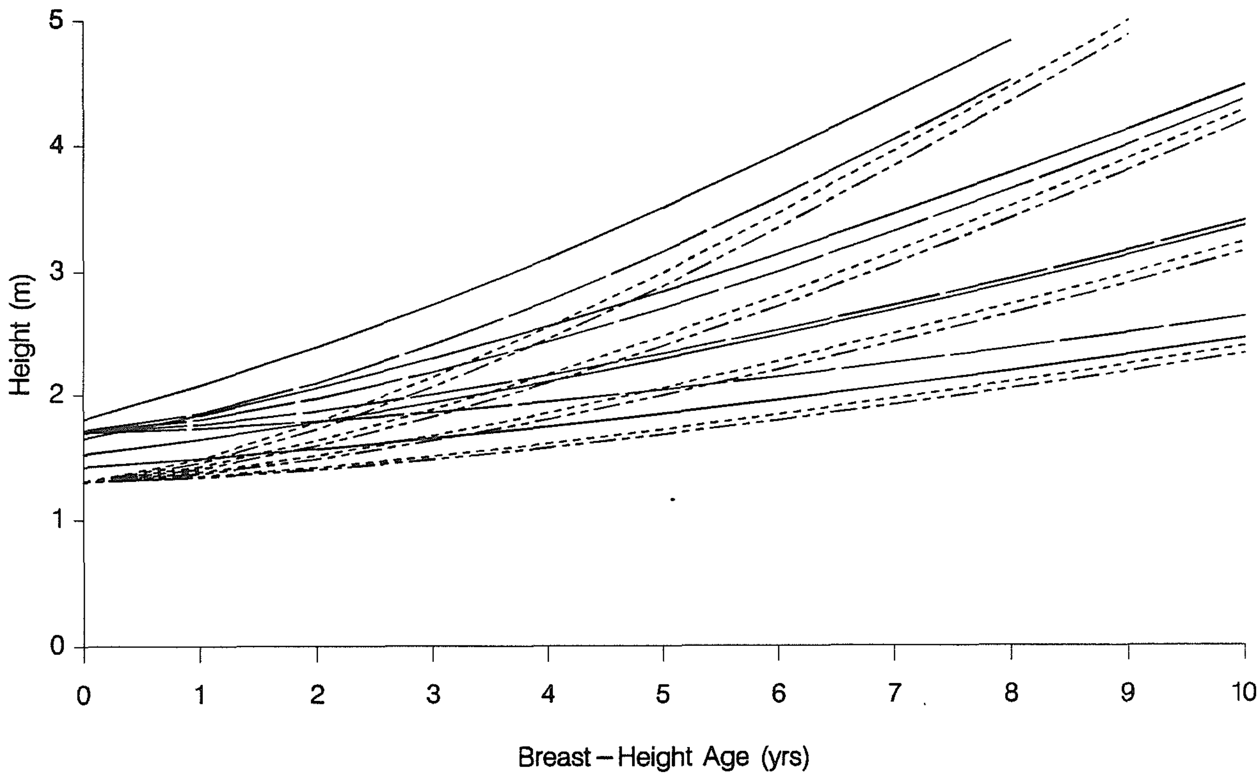

Figure 7. Formulated height-age curves at early ages for the age-adjusted model (solid line), fixed-intercept model (dashed line), and the floating-intercept model (dot-dashed line).

Comparison with Goudie's Curves

The height-age curves formulated with the age-adjusted logistic function give similar estimates of height to Goudie's (1984) curves between about 10 and 80 years breast-height age (Figure 8). The new height-age model predicts greater heights than Goudie's model below 10 years of age and after 80 years of age. Goudie's curves under-predict the height of the 109 stem-analysis plots below about 15 year breast-height age and for all ages after 50 years (Figure 9). Thus the new curves should more accurately reflect the height-growth patterns of interior spruce in B.C.

Years To Breast Height

The number of years to reach breast height (1.3 m) was not strongly related to site index (Figure 10). This is typical for shade-tolerant species where early growth is also affected by non-site factors such as competition from brush, weeds, and over-topping from the canopy of taller trees. The number of years to reach breast height (*yrbh*) can be estimated by:

$$yrbh = yrst + 0.38 + 117.34 / SI \quad [5]$$

where *yrst* is the estimated number of years to reach stump height (0.3 m) (4 years was used in this analysis), and *SI* is site index (dominant height at 50 years breast height age). This equation had a RMSE = 2.32 m and R^2 = 0.43.

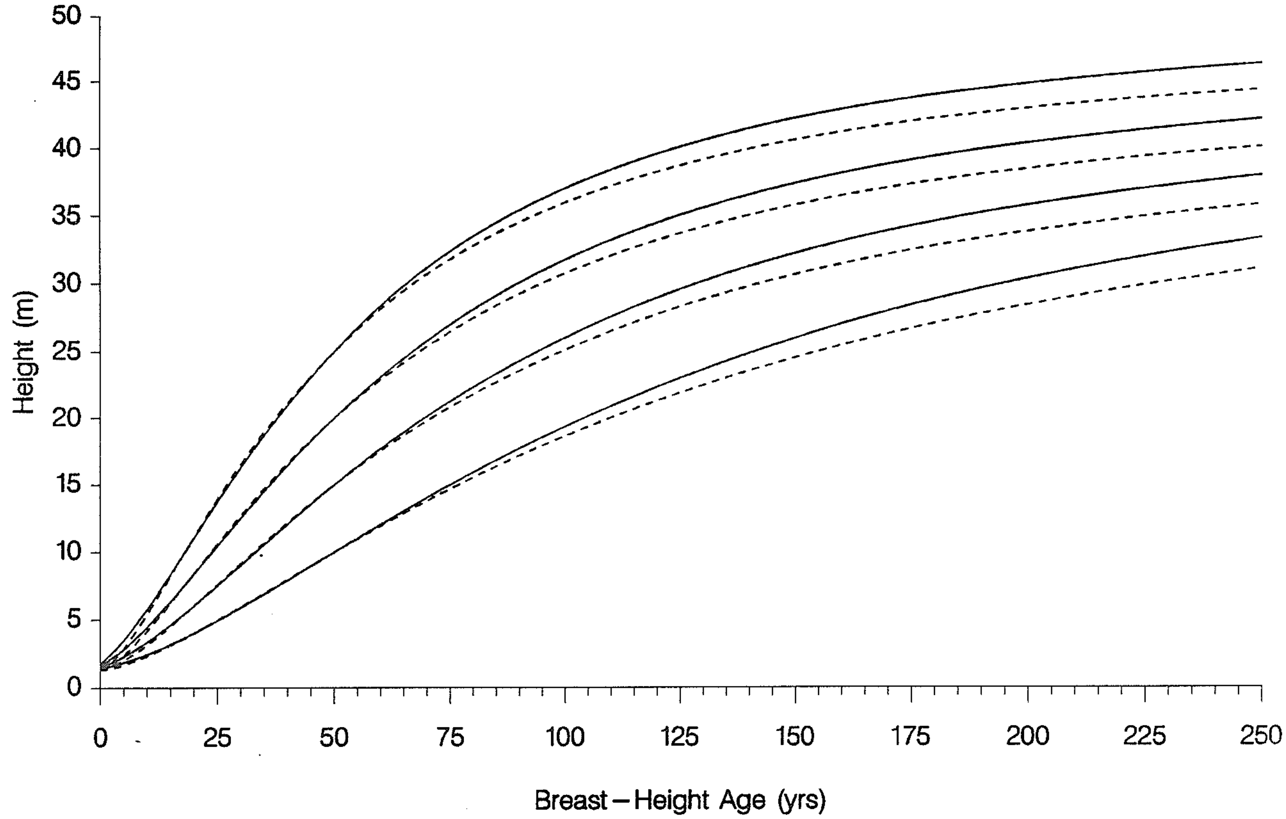

Figure 8. The height-age curves formulated with the age-corrected model (solid) and Goudie's (1984) curves (dashed line).

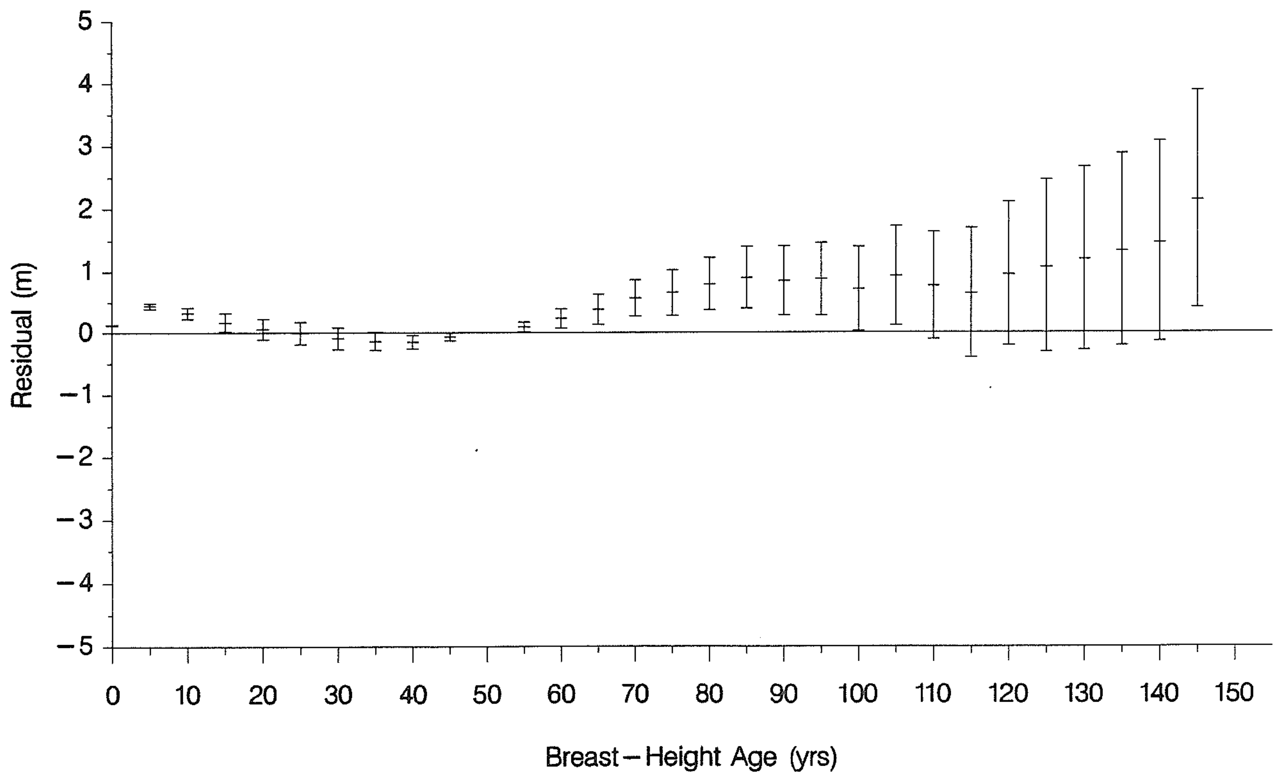

Figure 9. Mean and two standard errors of height residuals from Goudie's (1984) curves compared to the 109 stem-analysis plots.

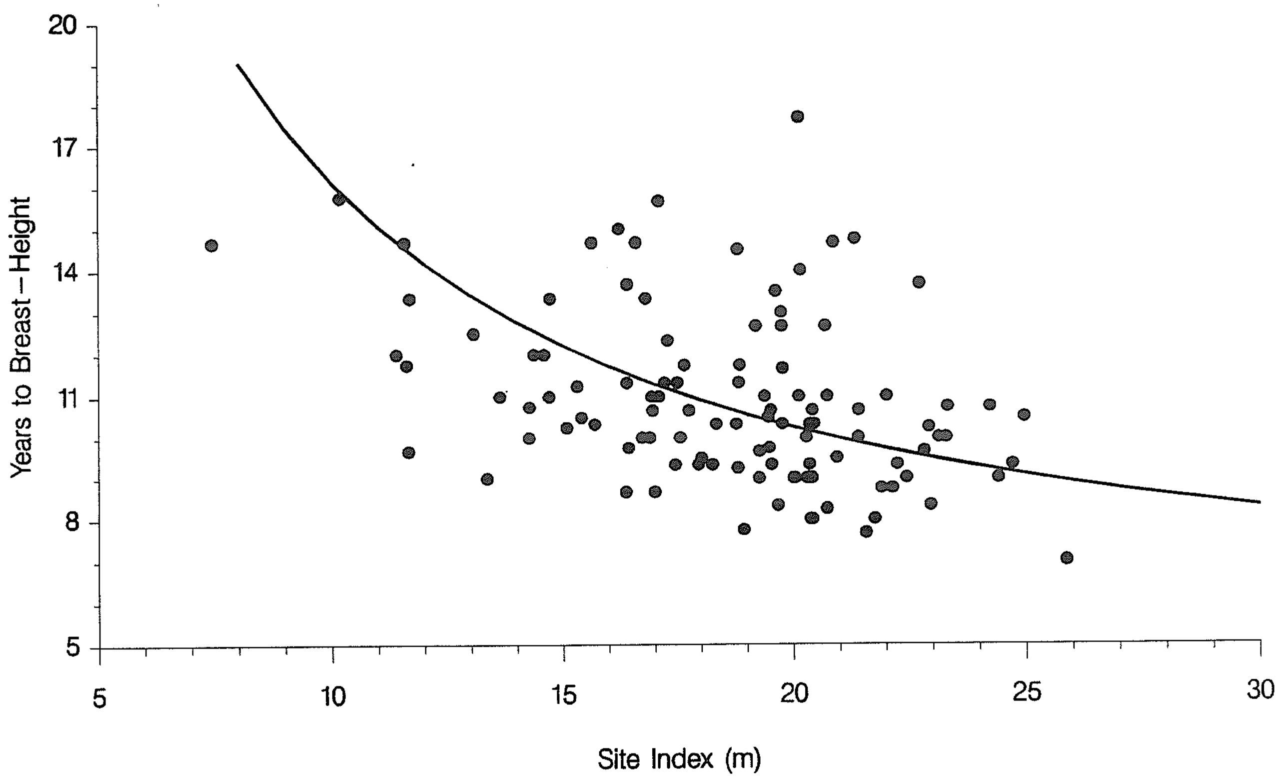

Figure 10. Number of years to reach breast height for the 109 interior spruce sample plots. Predicted values are shown by the solid line.

Site-Index Curves

The conditioned logistic model was selected as giving the best fit to the site-index data. The formulated model (Figure 11) is:

$$SI = H \left[\frac{1 + e^{(0.8524 \ + \ 1.1175 \ln 50 \ + \ -1.4747 \ln H)}}{1 + e^{(0.8524 \ + \ 1.1175 \ln A \ + \ -1.4747 \ln H)}} \right] \qquad [6]$$

where *SI* is the site index (dominant height (m) at 50 years breast-height age), *H* is height (m); *A* is breast-height age (yrs), and A_{SI} is the reference age of the site index (50 years breast-height age).

This model showed a reasonable pattern of residuals over age (Figure 12) and site index (Figure 13). However, we do not recommend that this formulation is used for estimating site index. The site index of dominant, undamaged, free-growing interior spruce should be estimated with the height-age equation. This can be done using iterative techniques or the table in Appendix III. The iterative method gives estimates with slightly less bias than the site index equation within the range of the data (Figure 14).