

# Height-Age and Site-Index Curves for Lodgepole Pine in British Columbia

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## Executive Summary

Site index is the most important predictor variable in estimating growth and yield of natural and managed stands of lodgepole pine (*Pinus contorta* var. *latifolia* Dougl.) and accurate estimates of site index are needed for long-range planning and stand-level decisions. The Ministry of Forests initiated a program in the mid 1980s to replace the formerly used height-age curves that were developed from temporary sample plot data. New curves for lodgepole pine had recently been developed from stem-analysis data collected primarily from Alberta (Goudie 1984), however, additional data were collected from B.C. because of concern that height-growth patterns may be somewhat different in Alberta. The additional data were collected by Industrial Forestry Service (IFS) under contract to the Ministry. The objective of the analyses reported in this document was to formulate new height-age and site-index curves based on these new stem-analysis data.

The data were collected in 1988-89 from 59 plots located in the SBS, IDF, ICH, MS, and ESSF Biogeoclimatic zones. The plots ranged in breast-height age from 50 to 210 years. Two plots were greater than 130 years of age. Site index ranged from 6-28 m with 42 of the plots (71%) in the 15- and 20-m site-index classes. Three variations of the conditioned logistic function were developed and tested for fit to the height data and for extrapolation beyond the range of site index and age. Three nonlinear models were tested for predicting site index as a function of height and age.

The conditioned logistic function with an age-adjustment parameter was selected as providing the best fit to the height data. This model should provide good estimates of height for the range of age and sites found in B.C. The curves may slightly under-predict height at older ages and on higher sites. The model was virtually unbiased for height across the range of age, however, was biased at breast-height age zero. This diminished rapidly and was negligible at age 5. The other models were biased for 10 years or more above breast height. The formulated height-age curves were almost identical after index age to the currently used curves (Goudie 1984). Goudie's curves showed a small bias at early ages when compared to the plot data. This was probably the result of the model and does not indicate that the data or pattern of height growth were different. The number of years to reach breast height averaged about 7 years and ranged from 4 to 14 years. An equation developed to predict years to reach breast height as a function of site index for these plots located in natural stands was very similar to the equation for planted stands given in the recent yield tables for managed lodgepole pine (Goudie *et al.* 1990).

The conditioned logistic function was selected as providing the best predictions of site index. The model had relatively good precision and the bias did not exceed 0.5 m from age 10 to 115 years breast-height age and for site-index classes 5- to 25-m. The height-age curves gave equally good predictions when used iteratively to estimate site index and have the advantage of being compatible with height and age. However, the site-index curves have the advantage of providing a closed-form model where site index is predicted directly.

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## INTRODUCTION

Lodgepole pine (*Pinus contorta* var. *latifolia* Dougl.) grows throughout the interior of British Columbia. It is one of the most widely distributed pines in North America and ranges from the Yukon Territory south to Utah and Colorado States, and from the west coast of British Columbia east to the Alberta foothills of the Rocky Mountains. Lodgepole pine regenerates vigorously to pure stands after wildfire. These stands are gradually invaded by other more shade tolerant species if not destroyed by fire and are sometimes found mixed with spruce and aspen.

Lodgepole pine is probably the most important timber species in the interior of British Columbia with an estimated 1.9 billion m<sup>3</sup> of merchantable timber (31% of the total volume of commercial species).<sup>1</sup> The total area of forest types in the interior where lodgepole pine is the leading species is estimated at about 14 million ha (38% of total forest area). In 1989, approximately 38% of the total wood volume harvested in the interior was lodgepole pine (21.5 million m<sup>3</sup>), followed by spruce at approximately 29% of the total volume (16.3 million m<sup>3</sup>) (Ministry of Forests 1991).

Site quality evaluation is a very important part of the management of lodgepole pine in B.C. The estimated site index has a dramatic affect on growth and yield predictions in timber supply analyses and in comparing and evaluating silvicultural treatments. Thus it is very important that accurate methods are used in estimating site index. Site index for lodgepole pine and other species in B.C. has traditionally been estimated with height-age curves (commonly called site-index curves). Anamorphic height-age curves were developed for lodgepole pine from observations of height and age from 2074 trees collected in temporary sample plots located in previous Province-wide inventories (Ministry of Forests 1981). These curves were used in British Columbia until the mid 1980s when new curves were developed from stem-analysis data.

The site-index curves currently used for lodgepole pine in British Columbia were developed using stem-analysis data from 134 trees from Alberta and 54 trees from B.C. (Goudie 1984). These curves were developed to provide a more realistic description of the height growth of dominant trees for modelling the growth and yield of lodgepole pine with the TASS model (Tree And Stand Simulator - Mitchell 1975). Concurrently, the Ministry of Forests initiated a program to replace the anamorphic site-index curves that were used for all species in the Province (Ministry of Forests 1981). The intent was to use stem-analysis data from B.C. to develop curves that reflected actual height-age growth patterns and provided more precise and accurate estimates of height and site index. Although curves had been developed from stem-analysis data (Goudie 1984), the extreme importance of lodgepole pine and some concern about the high proportion of data from Alberta led to further sampling in the interior of B.C.

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<sup>1</sup>Summary of Forest Inventory statistics provided by Anthony Britneff, Ministry of Forests, Inventory Branch, December 1992.

The data collected for developing the lodgepole pine height-age and site-index curves in this report were done under contract with Industrial Forestry Service (IFS) in 1988 and 1989. The objective of the analysis phase of the project reported here was to formulate new height-age and site-index curves using these data. The intent was to develop curves that would provide accurate estimates of height and site index for the management of lodgepole pine in the interior of B.C.

## METHODS

### Plot Data

Stem-analysis data were collected from 59 plots throughout the range of lodgepole pine in the interior of the Province. The data were collected in 1989 and 1990 under two different contracts with IFS of Prince George, B.C. (IFS contract numbers 890355 and 891090, respectively). The objective for the first contract was to sample older stands (greater than about 100 years of age) across the range of sites in the SBS and SBPS Biogeoclimatic zones. The objective of the second contract was to sample older stands in the Nelson and Kamloops Forest Regions. Twenty-eight plots were sampled in the SBS Biogeoclimatic zone in the Prince George, Prince Rupert, and Cariboo Forest Regions in the first contract. An additional 31 plots were sampled in the ESSF, ICH, IDF, MS, and SBS Biogeoclimatic zones in the Kamloops and Nelson Forest Regions in the second contract (Table 1).

Table 1. Number of sample plots by Forest Region, Biogeoclimatic zone, and year of sampling.

Forest Region	Biogeoclimatic Zone (and year of sampling)							Total
	SBS			MS	IDF	ICH	ESSF	
	1989	1990	Total	1990	1990	1990	1990	
Cariboo	8	0	8	0	0	0	0	8
Kamloops	0	3	3	8	2	0	2	15
Nelson	0	0	0	4	5	7	0	16
Prince George	16	0	16	0	0	0	0	16
Prince Rupert	4	0	4	0	0	0	0	4
Total	28	3	31	12	7	7	2	59

Descriptive information was recorded for each 0.04 ha sample plot including the species and diameter of all trees. Each plot was classified to the BEC subzone level, however, classification to the site level was not done on these plots. Elevation, slope, and aspect were recorded for all plots, and latitude and longitude were recorded for the plots sampled in 1990. Four site trees were selected from each sample plot as the largest diameter,

dominant trees showing no signs of damage or stem defect. The site trees were felled and disks cut for stem analysis at stump height (0.3 m), breast height (1.3 m), and at nine equally spaced intervals above breast height. The disks were transported to a laboratory where the number of rings were counted for each section. Radial growth measurements were also taken at 5-year intervals for two average radii from each disk (the radial growth data were not analyzed in this project).

## Plot Height-Age Curves

Height-age curves were constructed for each site tree based on total age and breast-height age. The height where the stem analysis disks were cut from the tree was corrected for bias using the method of Carmean (1972). This process adjusts the height that corresponds to the age of the disk to coincide (on average) with the node that marks the end of height growth for that year. Height-age curves were then constructed for each tree based on total age to determine the number of years to reach breast height. The total age of each tree was estimated by adding 2 years to the age of the disk taken at stump height. The number of years for each tree to reach breast height was computed as the difference in age between the breast-height disk and the total age of the tree. The total-age curves were examined for abnormal growth patterns of trees within plots indicating damage or suppression. One tree was discarded from the 1990 data because it was dramatically different from the other trees in the plot.

Curves based on breast-height age were constructed to represent the average height growth of the four dominant trees in each plot. The height of each site tree was determined at each 5-year interval of breast-height age through linear interpolation of the original, corrected height-age curves. The plot-average curve was then constructed by averaging the height of the site trees at each 5-year interval. The average curve was based on all site trees in the plot, thus the total age of the average curve was limited by the age of the youngest site tree. Site index for each plot was the height of the average curve at 50 years breast-height age. The average total breast-height age of the 59 plots was 97 years and ranged from 50 to 210 years (Figure 1). The average site index was 17.2 m and ranged from 5.7 to 27.8 m (Table 2).

Table 2. Number of sample plots by 5-m site-index class and Biogeoclimatic zone.

BEC Zone	Site-index Class (m)						Total
	5	10	15	20	25	30	
SBS	2	4	6	14	5	0	31
MS	1	1	8	1	1	0	12
IDF	0	1	2	4	0	0	7
ICH	0	0	1	4	1	1	7
ESSF	0	0	2	0	0	0	2
Total	3	6	19	23	7	1	59

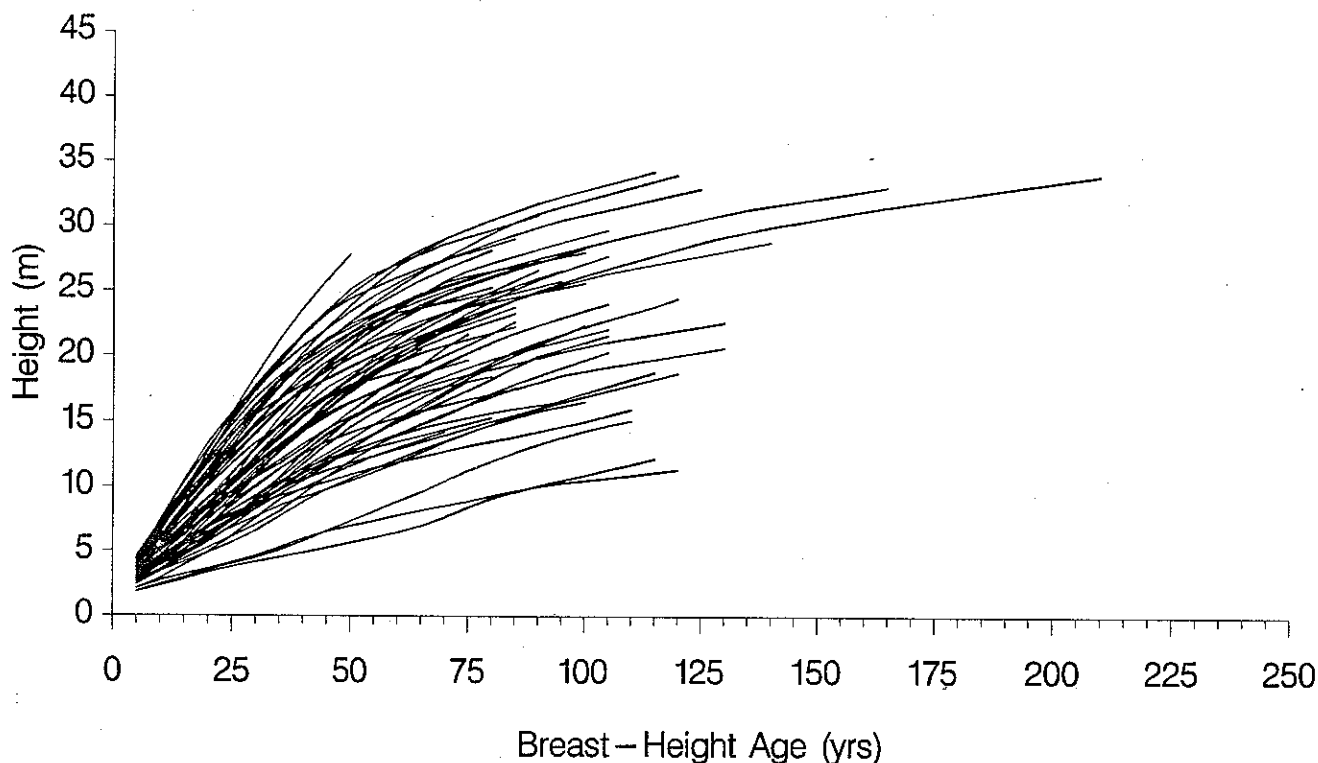


Figure 1. Breast-height age curves for the 59 lodgepole pine sample plots.

### Formulated Height-Age Curves

Many height-age models based on modifications of the 5-parameter Chapman-Richards function (Ek 1971) and the modified logistic function (Monserud 1984) were tested for describing the height-age patterns. These included both conditioned and unconditioned versions of the models. Initial results indicated little difference between the two families of models or between the conditioned and unconditioned versions. Thus the conditioned logistic function was chosen for further development and modification. The logistic type functions have desirable properties over the Chapman-Richards function such as more flexibility in early years immediately above breast height. They also show more sustained height growth at older ages when extrapolated beyond the age of the data where the growth rate of the Chapman-Richards functions often diminishes rapidly. The conditioned version of the logistic function was chosen because differences from the unconditioned version were small, and the conditioned function has the desirable property of predicting site-index height at index age.

Initial results also indicated that the models were biased at young ages. This resulted in developing and testing versions of the conditioned logistic function having a floating intercept. Also the models were not flexible enough to adequately describe height growth above and below index age. A segmented logistic function fit separately to the data above and below index age was tested for describing these patterns separately. The model gave

very good results, but could not be smoothed at index age to show a continuous growth rate and thus was not considered for further testing.

The three variations of the conditioned logistic function that were selected for final evaluation were: the fixed-intercept model

$$[1] \quad H = 1.3 + (SI - 1.3) \left( \frac{1 + e^{(b_1 + b_2 \ln A_{50} + b_3 \ln (SI - 1.3))}}{1 + e^{(b_1 + b_2 \ln A + b_3 \ln (SI - 1.3))}} \right)$$

the floating-intercept model

$$[2] \quad H = (b_1 + b_2 SI) + (SI - (b_1 + b_2 SI)) \left( \frac{1 + e^{(b_3 + b_4 \ln A_{50} + b_5 \ln (SI - (b_1 + b_2 SI)))}}{1 + e^{(b_3 + b_4 \ln A + b_5 \ln (SI - (b_1 + b_2 SI)))}} \right)$$

and the age-adjusted model

$$[3] \quad H = 1.3 + (SI - 1.3) \left( \frac{1 + e^{(b_1 + b_2 \ln (A_{50} + b_4) + b_3 \ln (SI - 1.3))}}{1 + e^{(b_1 + b_2 \ln (A + b_4) + b_3 \ln (SI - 1.3))}} \right)$$

where  $H$  is height (m);  $b_i$  are regression coefficients;  $SI$  is the site index (dominant height (m) at 50 years breast-height age),  $A$  is breast-height age (yrs), and  $A_{50}$  is the reference age of the site index (50 years breast-height age).

The models were fit to the average curves for the 59 plots up to 130 years of age which included 1115 observations of height at 5-year intervals of breast-height age. Only 2 plots exceeded 130 years of age and were restricted to medium sites (Figure 1). Sufficient numbers of plots were not available across the various Biogeoclimatic classification units to examine differences in height-age patterns by ecosystems.

Evaluation criteria used to select the final model were the statistical fit to the data and the pattern of extrapolation beyond the range of age and site index. Primary evaluation was based on graphical comparison of residual scatterplots and summary statistics. Plots of height residuals over breast-height age and 5-m site-index classes were examined for bias and variance. The formulated curves were then compared to the height-age curves currently used in British Columbia (Goudie 1984).

## Formulated Site-Index Curves

Several linear and nonlinear models were tested for predicting site index as a function of average dominant height and breast-height age. The linear models were developed using Dahms' method of constructing potential predictor variables. The nonlinear models included:

the conditioned logistic function

$$[4] \quad SI = H \left[ \frac{1 + e^{(b_1 + b_2 \ln A_{50} + b_3 \ln H)}}{1 + e^{(b_1 + b_2 \ln A + b_3 \ln H)}} \right]$$

the age-adjusted logistic function

$$[5] \quad SI = H \left[ \frac{1 + e^{(b_1 + b_2 \ln(A_{50} + b_4) + b_3 \ln H)}}{1 + e^{(b_1 + b_2 \ln(A + b_4) + b_3 \ln H)}} \right]$$

and Payendeh's (1974) model based on the Chapman-Richards function

$$[6] \quad SI = b_1 H^{b_2} (1 - e^{-b_3 A})^{b_4 H^{b_5}}$$

where  $SI$  is the site index (dominant height (m) at 50 years breast-height age),  $H$  is height (m),  $b_i$  are regression coefficients, and  $A$  is breast-height age (yrs).

The site-index models were fit to the data between 10 and 120 years of age. Height is very poorly related to site index at very early ages, thus the data for 5 years of age were not included in the analysis. There were few plots above 120 years of age thus these data also were not included in the analysis. The models were evaluated using graphical techniques across the range of age and site index for the pattern and magnitude of bias. The model showing the least bias was selected for predicting site index from the average dominant height and breast-height age.

## RESULTS AND DISCUSSION

### Height-Age Curves

#### Formulation

The model selected as providing the best description of the height-growth patterns of dominant lodgepole pine was the age-adjusted conditioned logistic function:

$$[7] \quad H = 1.3 + (SI - 1.3) \left[ \frac{1 + e^{(9.1167 - 1.6512 \ln(50 + 4.8189) - 0.9488 \ln(SI - 1.3))}}{1 + e^{(9.1167 - 1.6512 \ln(A + 4.8189) - 0.9488 \ln(SI - 1.3))}} \right]$$

where  $H$  is dominant height (m);  $SI$  is the site index (dominant height (m) at 50 years breast-height age), and  $A$  is breast-height age (yrs). The formulated curves are shown in Appendix I and predicted heights are given in Appendix II.

The age-adjusted conditioned-logistic function (Eqn. 3) selected as the final model was similar to the fixed-intercept (Eqn. 1) and the floating-intercept model (Eqn. 2) in the

overall average fit to the data (Table 3). These models also gave very similar extrapolations at higher site indices and older ages (Figure 2). The difference at older ages was relatively small and thus the effect on growth and yield modelling should also be small. Height-growth patterns of the two plots greater than 130 years of age and those greater than about 100 years suggest that the final curves may slightly under-predict height at older ages (Figure 3). Very little confidence can be placed in the patterns displayed by only two plots, thus more data are needed to precisely describe height growth at these older ages.

Table 3. Regression coefficients and root mean squared error (RMSE) of the three height-age models.

Statistic	Height-Age Model		
	1	2	3
RMSE	0.8771	0.8568	0.8631
$b_1$	8.1476	1.7916	9.1167
$b_2$	-1.3241	0.0412	-1.6512
$b_3$	-1.0457	8.7952	-0.9488
$b_4$		-1.5302	4.8189
$b_5$		-1.0456	

The three models were very similar in the overall fit to the data and extrapolation, however, differences were more pronounced at early ages. The fixed-intercept model (Eqn. 1) showed significant bias below index age and under-predicted the height of the sample plots (Figures 4 and 5). This bias at early years was somewhat reduced by allowing the intercept to float (Eqn. 2). However, this modification was not adequate to completely eliminate the bias and resulted in a small over-prediction of height at 5-years breast-height age. The final model with the age adjustment gave the best pattern of residuals. Bias was dramatically reduced at 5-years breast-height age and the mean residuals were not significantly different from zero at all 5-year intervals of age. However, the variation in plot heights at ages greater than index age was considerable as indicated by the 2-standard-error bars (Figure 5). The general trends of the residuals shown in Figures 4 and 5 did not vary by site class for the three models.

The pattern of residuals by 5-m site-index class was the same for all three models (Figure 6). Generally, they were relatively unbiased for site classes 5-25 m, but showed an over-estimate of height in the 30-m site-index class. Only one plot was in the 30-m site class, thus little confidence can be placed in the indicated bias.

### Comparison with Other Curves

The final height-age curves were very similar at older ages to the height-age curves given by Goudie (1984) for dry ecosystems (Figure 7). Goudie's curves for wet ecosystems indicate greater height-growth at older ages. This is consistent with the two older plots in the data (Figure 3). Goudie fit the fixed-intercept conditioned logistic function (Eqn. 1) to

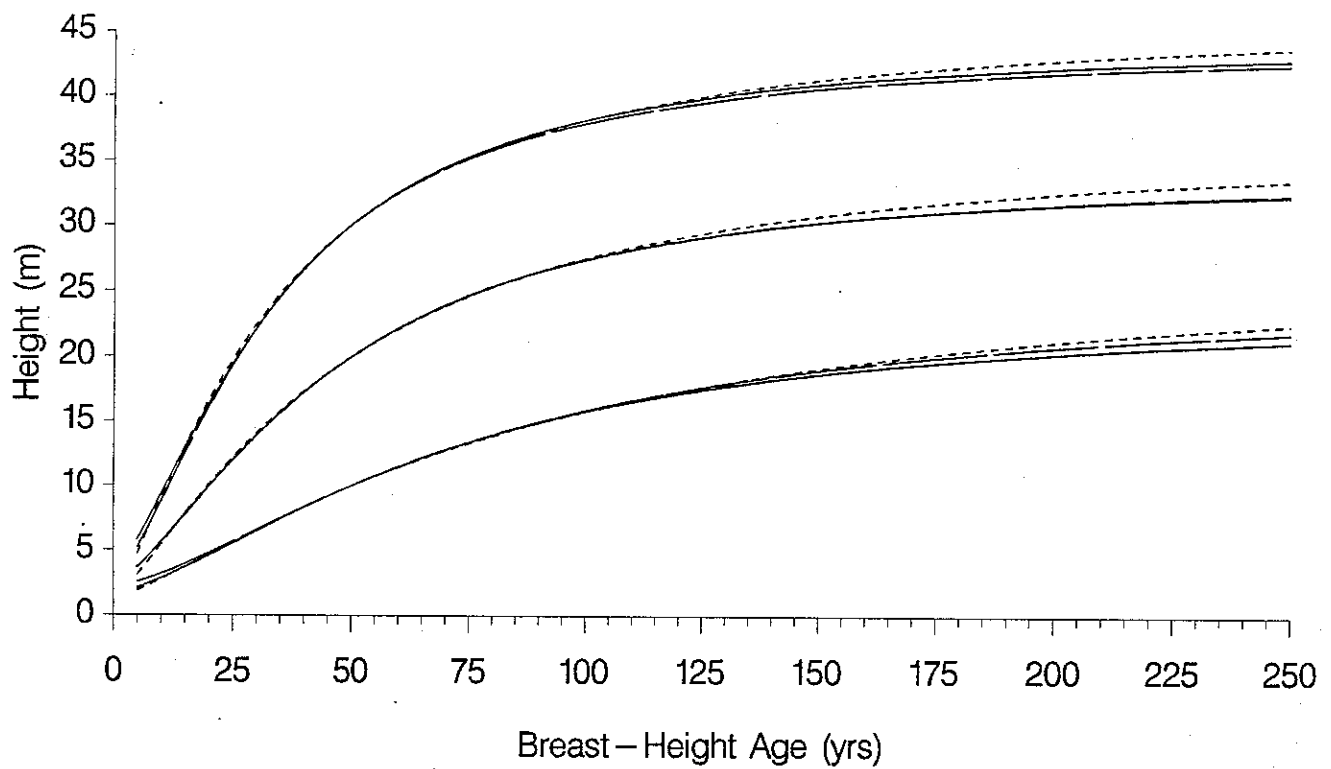


Figure 2. Height-age curves for the three conditioned logistic functions (solid: age-corrected model, short-dashed: fixed-intercept model, long-dashed: - floating-intercept model)

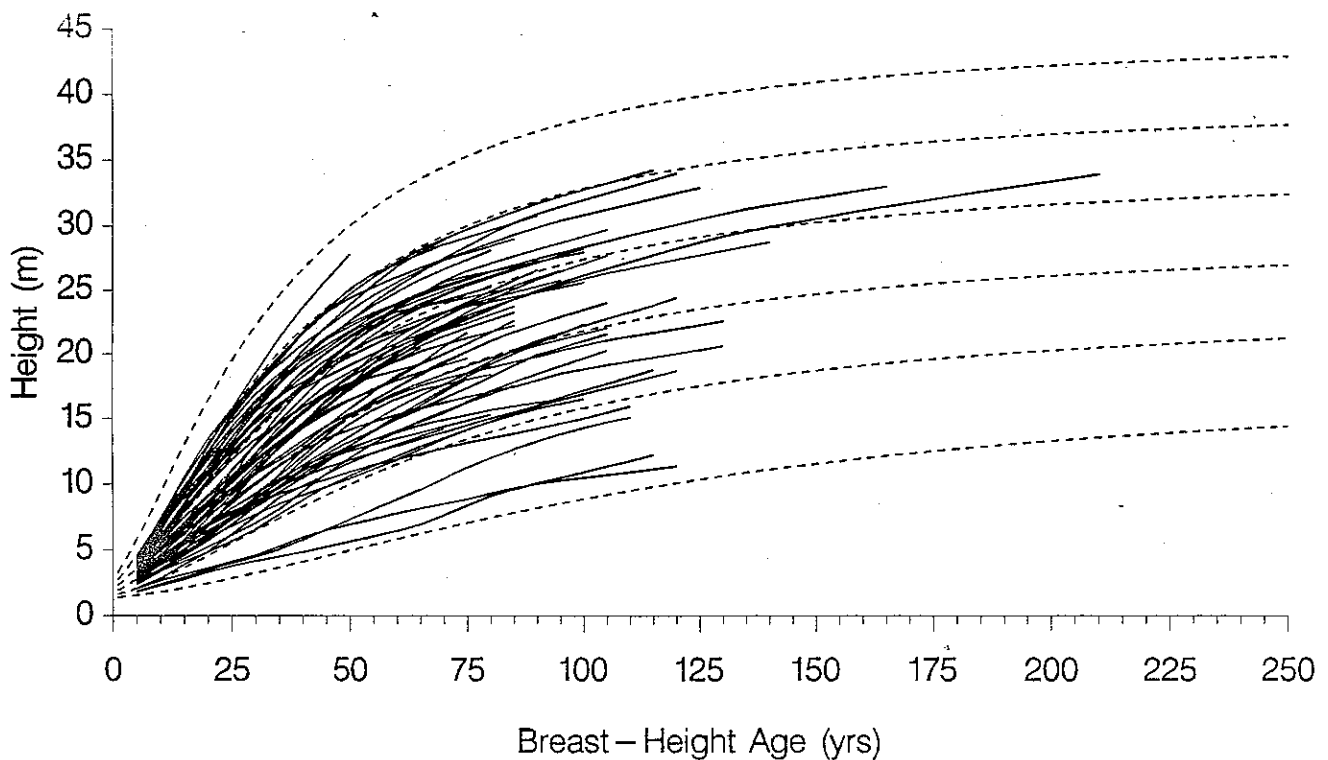


Figure 3. Formulated height-age curves for the final model (Eqn. 7) and the 59 stem-analysis plots.

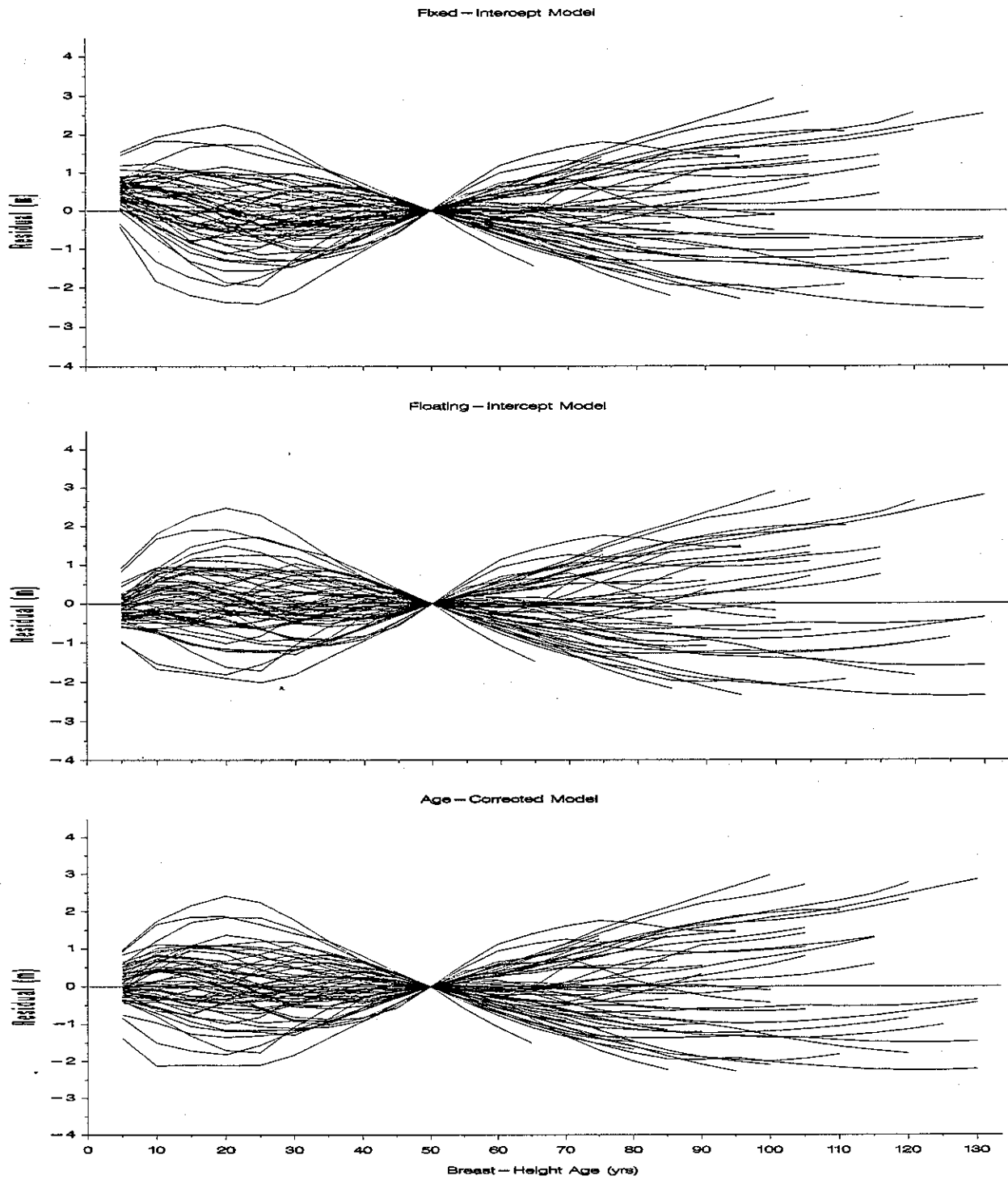


Figure 4. Spider plots showing residuals of height for the individual plots from the three models. Positive residuals indicate under-prediction of the model.

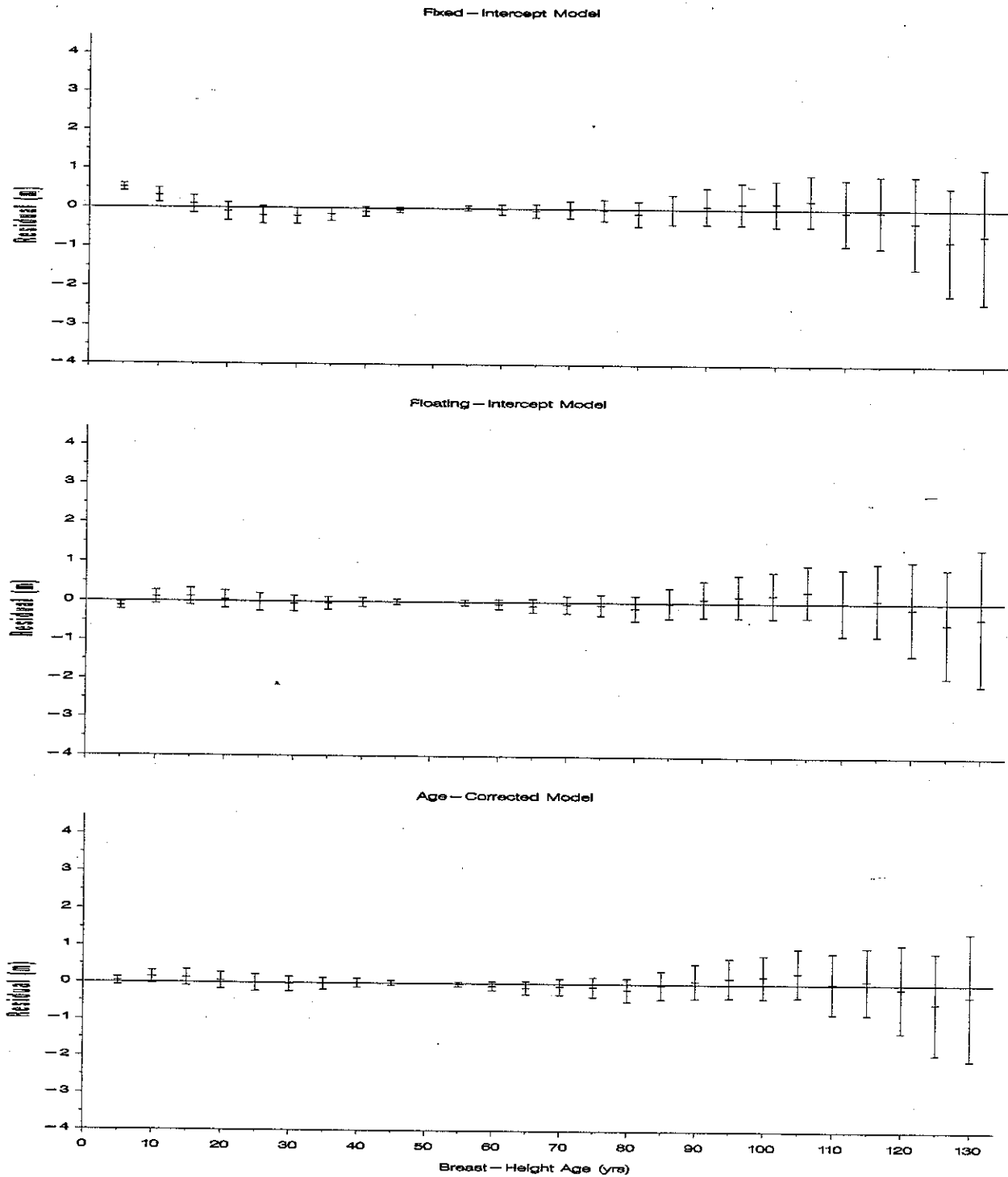


Figure 5. Mean and two standard errors of height residuals for the three models. Positive residuals indicate under-prediction of the model.

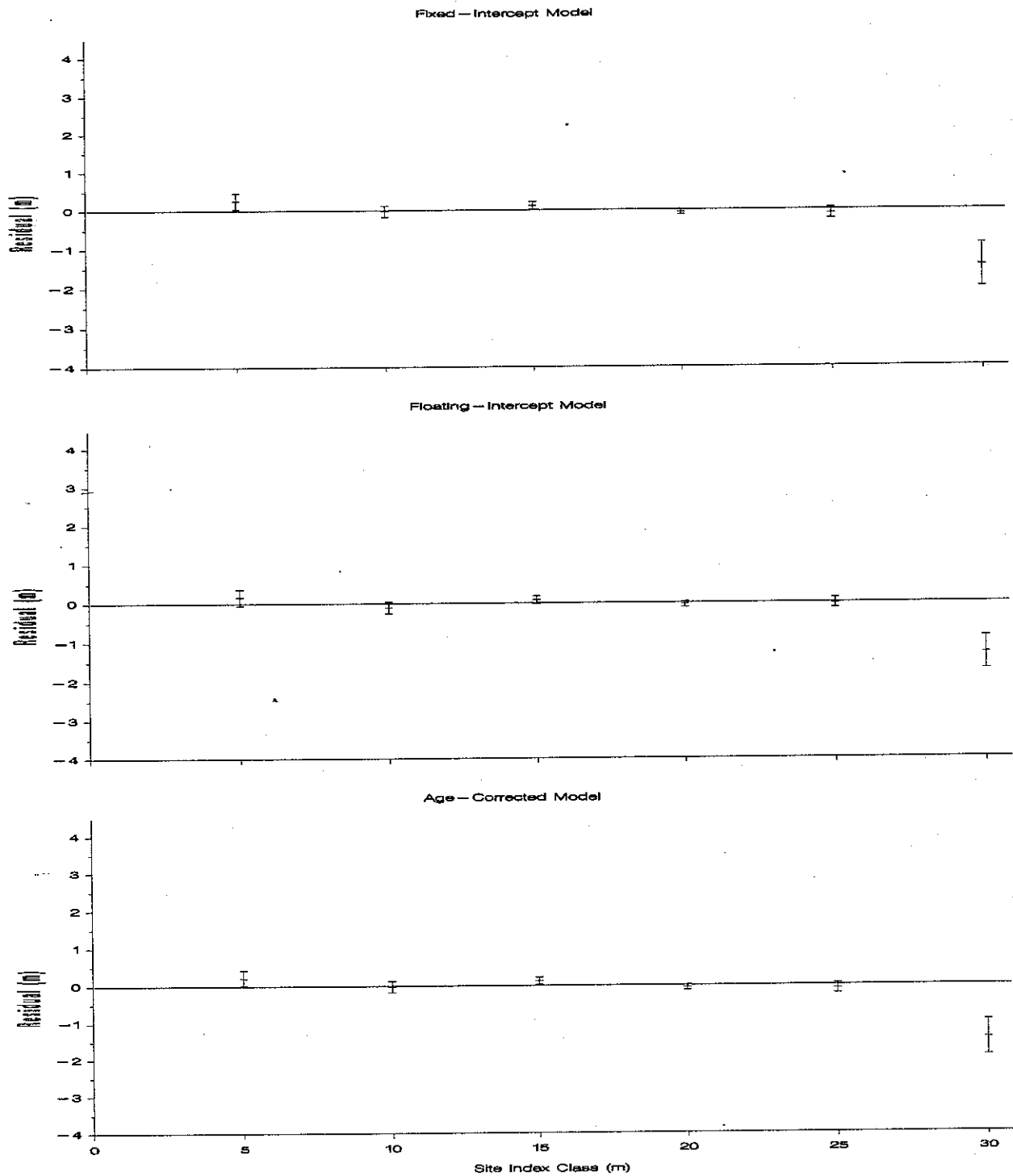
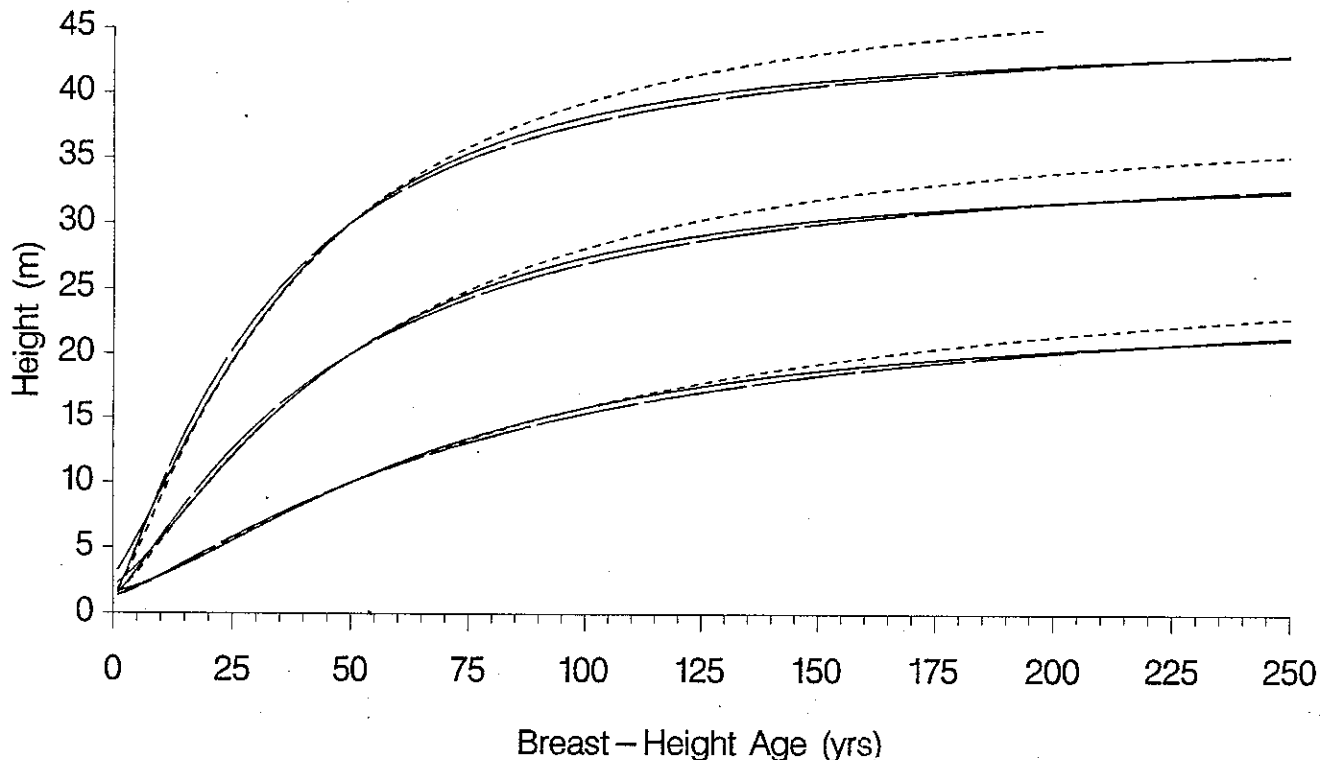


Figure 6. Mean and two standard errors for residuals of height by 5-m site index class. Positive residuals indicate under-prediction of the model.

stem-analysis data for 188 trees; 162 from dry ecosystems (86%) and 26 from wet ecosystems (14%). He recommended that the dry ecosystem coefficients be used if no ecological information was available because they represented the majority of data in the formulation. The root mean squared error (RMSE) in Goudie's formulation was 1.02 m compared to 0.86 m in our curves. Goudie's curves were fit to data based on 10-year intervals of age for individual trees. However, the fit may have shown a higher RMSE because they were based on individual trees and not averages as in our data.



**Figure 7.** Formulated height-age curves (solid) and Goudie's (1984) curves for wet (short dashed) and dry ecosystems (long-dashed).

Comparison of height-age patterns at early years showed small differences between Goudie's curves and these formulated curves. The residuals of actual plot height from the predicted height from Goudie's model showed the same pattern of bias as our residuals using the same model (Figures 8 and 5). These models gave a small under-prediction of height at age 5 and 10 years and slight over-prediction until index age. Thus the shape of Goudie's and our curves using Eqn. 1 appears to be an artifact of the model and does not reflect actual height-growth patterns. The residuals from actual plot heights after index age show the different shapes of Goudie's curves at older ages where the plot heights were slightly under-estimated with the dry curves and slightly over-estimated with the wet curves. The few observations at these older ages is reflected in the error bars that include zero for most 5-year intervals of age (Figure 8).

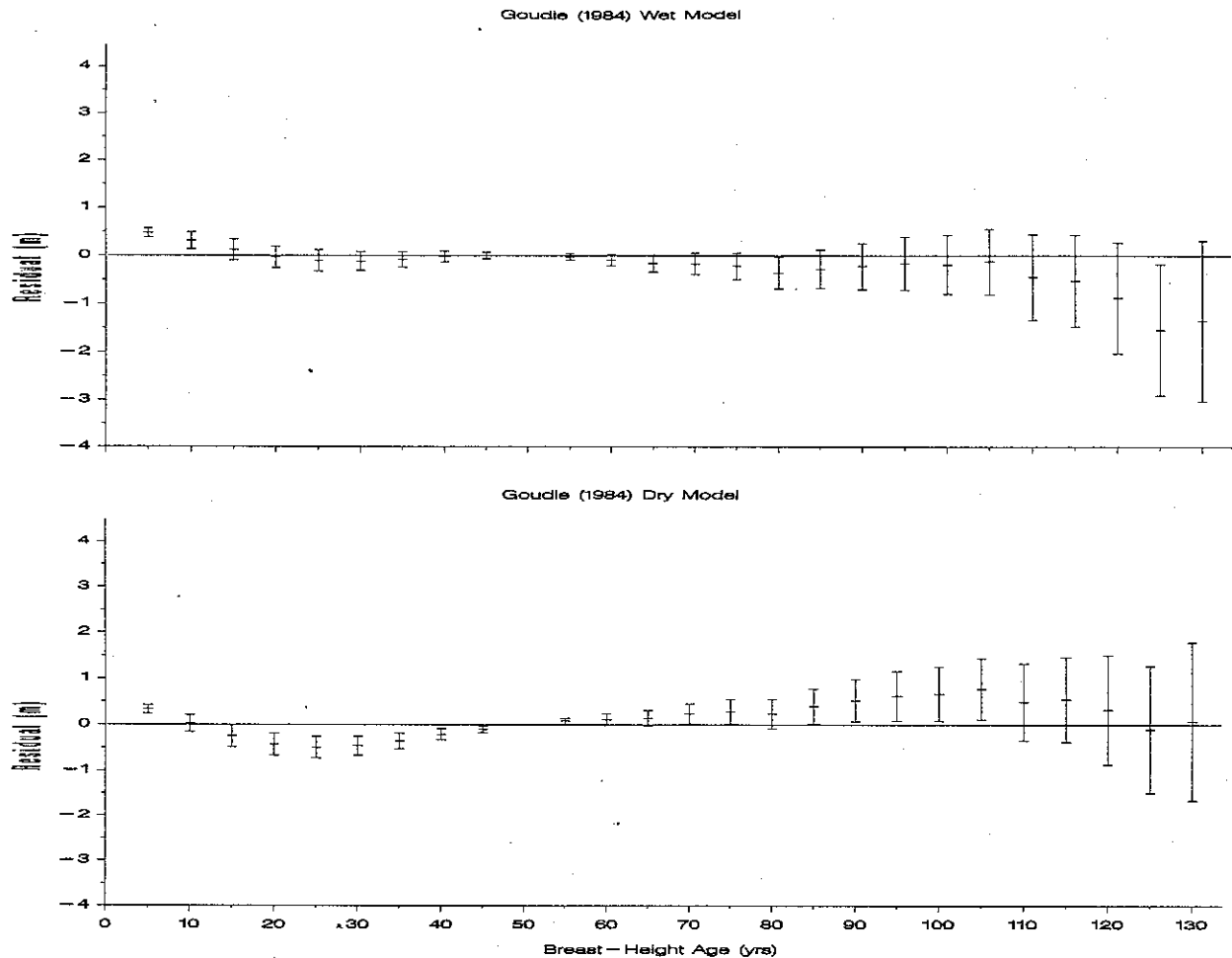


Figure 8. Mean and two standard errors of height residuals from Goudie's (1984) curves compared to the 59 stem-analysis plots.

#### ***Intercept of the Formulated Curves***

The modified logistic function used in this project and many other height-age curves based on breast-height age is conditioned to predict a height of 1.3 m at age zero. This has become somewhat of a convention, however, this gives biased estimates of height at early ages. The average height of trees at a breast-height age of zero is the expected value of the height of the first annual node *above* breast height - thus 1.3 m is the *minimum* height of this first node. The intercept should be higher than 1.3 m. Stem-analysis data for the development of height-age curves usually does not include annual observations of height. Thus this bias in conditioning the function to 1.3 m is not as evident because the first observations are usually at age 5 or 10 years. Annual height growth is greater on higher sites, thus the intercept (expected height at age zero) should be higher. This bias was evident in the analysis (Figure 5) which led to the testing the floating intercept model (Eqn. 2).

The bias in the fixed-intercept model (Eqn. 1) indicates that it was not flexible enough to assume a growth rate in the first 5 years that was consistent with the data (Figure 5). The floating-intercept model (Eqn. 2) decreased the bias and significantly improved the fit below index age. However, the age-adjusted model (Eqn. 3) achieved the best fit to the data. The effect of the age adjustment was to shift the origin of the function to the left (negative side of the age axis) to allow the model to achieve a growth rate that was more consistent with the data. This provided a much better fit at early years. However, the final height-age model (Eqn. 7) is biased at age zero.

Data were not available for the height of the site trees at age zero, however, we can estimate the average height of the first annual node above breast height. For all sites, the average height will be 1.3 m plus one-half of the expected annual height growth in the first year in which the tree reaches breast height (1.3 m). Height-growth rates of lodgepole pine change dramatically in early years, thus detailed stem-analysis would be required to obtain an accurate estimate for this phase of development. However, a rough guess of height-growth rates in this phase would be 0.3, 0.5, and 0.8 m for site indices 10, 20, and 30, respectively. Thus the intercepts would be  $1.3 + 0.3/2 = 1.45$  m,  $1.3 + 0.5/2 = 1.55$  m, and  $1.3 + 0.8/2 = 1.70$  m. The final height-age model (Eqn. 7) predicts intercepts of 1.6, 2.1, and 2.8 m for site indices 10, 20, and 30 m, respectively. Thus the final model is biased on sites above 10 m. The amount of bias increases with site index and is about 1.1 m for site index 30. Thus this model has a positive bias at age zero compared to the negative bias of the fixed-intercept model. However, the bias of the final model diminishes rapidly and gives unbiased estimates at age 5 whereas the fixed-intercept model still has a significant negative bias at age 5 and 10 years.

## Variation in Height-Growth Patterns

The analyses suggest the possibility of more than one pattern of height growth in lodgepole pine for the same level of site index. Visual inspection of the height-age curves (Figure 1) suggests that growth for some plots is sustained at a higher rate at older ages than other plots. Thus site index alone is not adequate to account for these differences in curve shape. Site index is an arbitrary point in time that is used to characterize height-age curves; however, it has no biological meaning or significance. Curve shape is related to site index through height-growth rate. This is the traditional definition of polymorphism, but there is no reason why site index should explain all the variation in height-growth development. There is increasing evidence for a variety of tree species that different height-growth patterns exist for the same level of site index. Thus site index alone cannot explain this variation.

Possible differences in the pattern of height-age development could be related to environmental or genetic factors. Few studies have related height-growth patterns to genetic origin, however, Buford and Burkhart (1987) and Nance and Wells (1981) reported no difference in the *shape* of height-age curves for different seed sources of loblolly pine in

the southeastern United States. Several preliminary studies have related the shape of height-growth curves to ecological factors. Monserud (1984) reported that height growth of interior Douglas-fir in the Inland Empire of the United States was sustained at a higher level on wetter ecosystems than drier sites at the same level of site index. Thrower and Goudie (1992) reported similar results for interior Douglas-fir in British Columbia. Goudie's (1984) formulated lodgepole pine curves for wet ecosystems in British Columbia and Alberta indicated sustained height growth compared to lower height-growth rates at older ages for dry ecosystems.

### Years to Breast Height

The average time for the trees in the 59 plots to reach breast height ranged from 4.25 to 14 years, and averaged 6.9 years with a standard deviation of 2.0 years. The number of years to reach breast height showed a fairly strong relationship to site index (Figure 9). This relationship is often very weak, especially for shade tolerant species where the growth rate of young trees is affected by vegetative competition. Lodgepole pine is usually very shade intolerant and will not survive severe shading. Thus the variation introduced from competing vegetation is not as pronounced as may be found in other species.

The number of years to reach breast height (*YRBH*) from seed can be predicted by:

$$[8] \quad YRBH = 2.68 + 65.55/SI$$

where *SI* is site index (dominant height (m) at 50 years breast-height age). This equation gives virtually the same results as the equation for planted lodgepole pine given by Goudie *et al.* (1990) where  $YRBH = 3.6 + 42.64/SI$ . The predicted number of years to reach breast height is the same as Goudie's equation for most sites greater than site index 10 m. The difference is larger at very low sites where at site index 5 Goudie's equation predicts 12 years and Eqn. 8 predicts 16 years.

### Site-Index Curves

The conditioned logistic model (Eqn. 4) was selected as providing the best predictions of site index as a function of average dominant height and breast-height age. The formulated model (Figure 10) is:

$$[9] \quad SI = H \left( \frac{1 + e^{(6.0925 + 0.7979 \ln 50 - 2.7338 \ln H)}}{1 + e^{(6.0925 + 0.7979 \ln A - 2.7338 \ln H)}} \right)$$

where *SI* is the site index (dominant height (m) at 50 years breast-height age), *H* is average dominant height (m), *b<sub>i</sub>* are regression coefficients, and *A* is breast-height age (yrs).

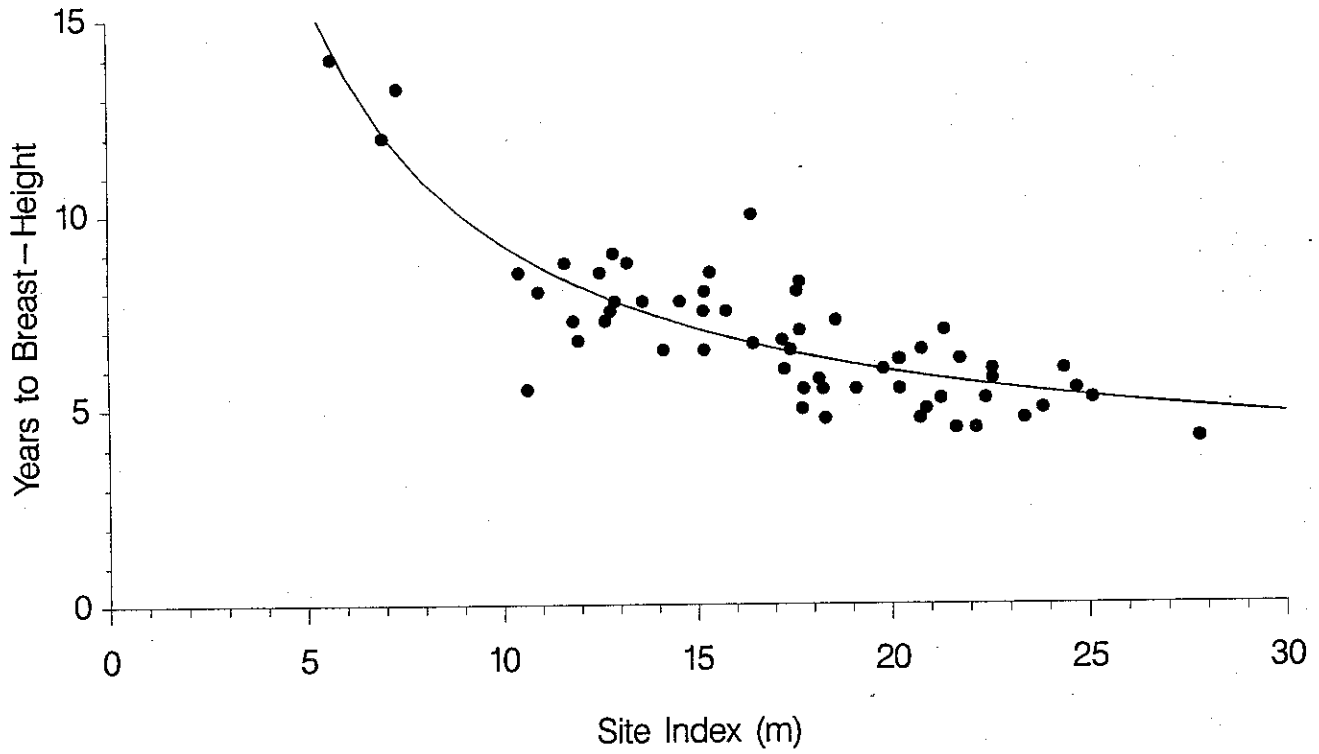


Figure 9. Number of years to reach breast height by site index.

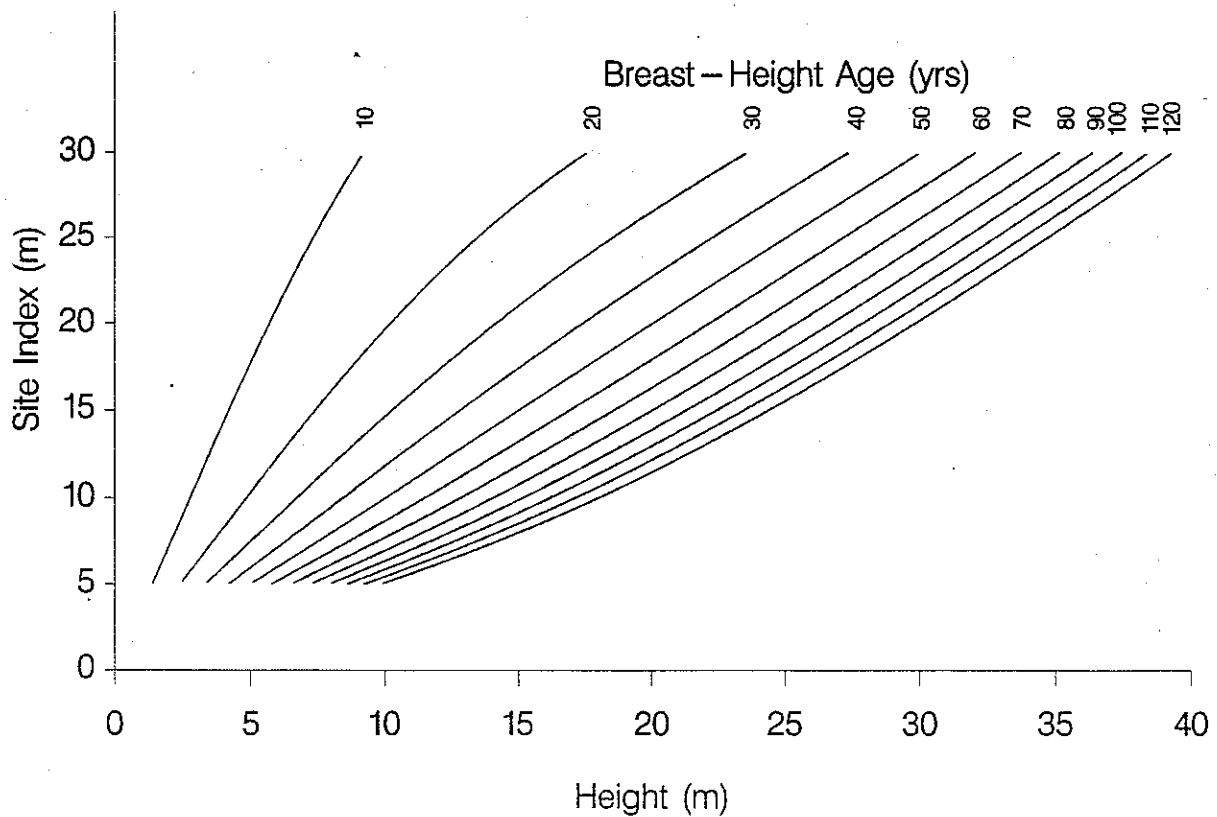


Figure 10. Formulated site-index curves.

The age-adjusted logistic function (Eqn. 5) showed the same pattern of residuals and RMSE of 1.12 m as the conditioned logistic function. Thus there was no advantage of including the extra parameter in the model. Payendeh's (1974) model (Eqn. 6) gave under-estimates of site index below index age and over-estimates after index age. This model gave a RMSE of 1.20 m and was not conditioned at index age and over-estimated site index at age 50. The linear models were strongly biased and did not achieve acceptable precision.

The final site-index model (Eqn. 9) showed a small bias over breast-height age that did not exceed 0.5 m except at age 120 where the sample size was small. The model under-estimated site index at age 10, over-estimated from age 15 to 50, and slightly under-estimated site index until age 120 (Figure 11). The model was also slightly biased by 5-m site-index class (Figure 12) but was less than 0.5 m for site classes 5 - 25 m. The bias was relatively large in the 30-m site-index class where the model under-estimated the actual site index by about 2.8 m. However, there was only one plot in the 30-m site-index class and thus little confidence can be placed in this estimate.

The formulated site-index curves did not provide more precise estimates than when site index was predicted with the height-age curves (Eqn. 7). The formulated height-age curves cannot be solved for site index. However, site index can be estimated by using iterative methods to determine the site index of the height-age curve that predicts a particular combination of height and age. Figure 13 shows the average residual from the stem-analysis plots at each 5-year interval of breast-height age for the site-index model (Eqn. 9), the height-age model (Eqn. 7), and Goudie's (1984) height-age model. All methods for estimating site index show an acceptable level of bias with average residuals less than about 0.75 m. Both the height-age and site-index models give very similar estimates, however, the major advantage of the site-index model is that it is a closed formulation and site index can be estimated directly. However, using the iterative technique with the height-age model ensures compatibility of height, age, and site index.

## **Additional Sampling Requirements**

### ***Ecological Variation***

The sample plots in this study were classified to the BEC subzone level. This is the level where differences in height growth patterns are most likely to occur, however, sufficient numbers of plots were not available at the same level of site index to test for possible differences (Table 2). We compared shapes of the curves for the 6 plots from the SBS and the 8 plots from the MS zones in the 15-m site-index class (Table 2). However, trends were not apparent and the variation within groups was as large as the variation between the two groups.

Differences in height-growth patterns must be tested at the same level of site index. Differences that are associated with site index are accounted for by polymorphic

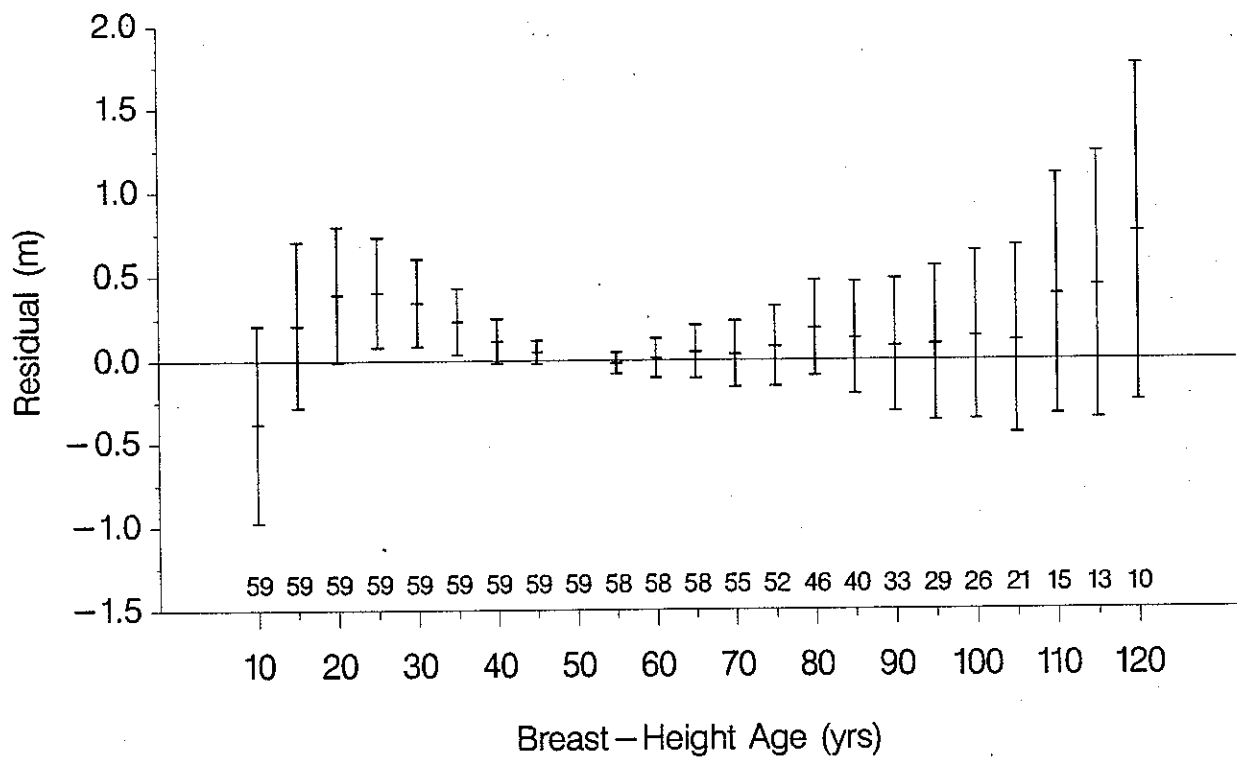


Figure 11. Mean and two standard errors of site index residuals by breast-height age for the formulated site-index model (Eqn. 9). The number of plots are given above each 5-year age interval.

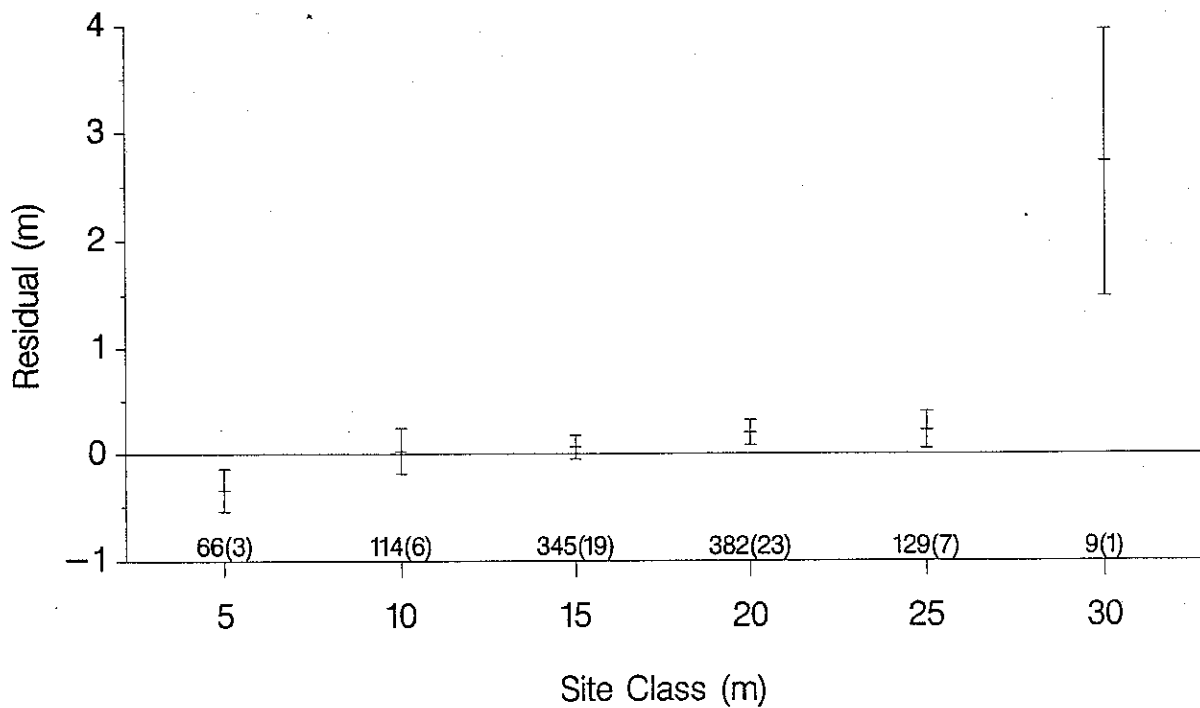


Figure 12. Mean and two standard errors of site index residuals by 5-m site-index class for the formulated site-index model (Eqn. 9). The number of observations and plots are given above each 5-year age interval.

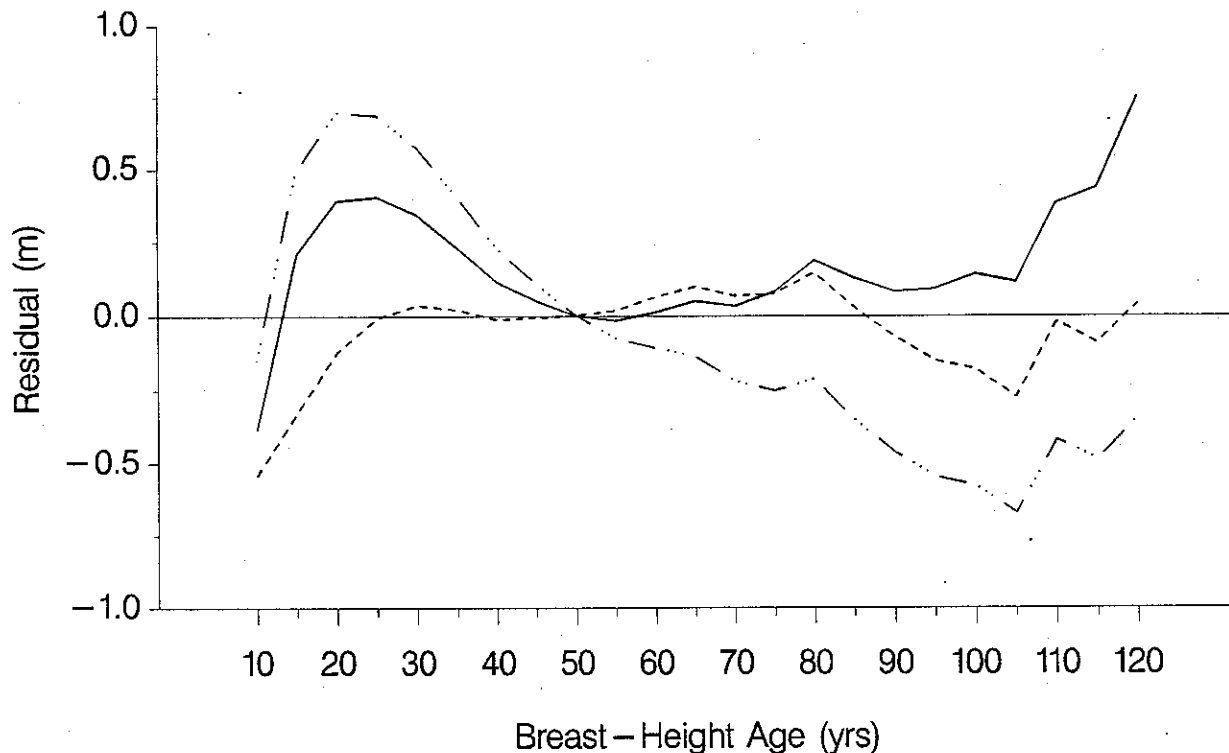


Figure 13. Average site index residuals from the stem-analysis data by breast-height age for the site-index model (Eqn. 9) (solid line), the height-age model (Eqn. 7) (dashed line), and Goudie's (1984) height-age model (dot-dashed line).

functions. Preliminary evidence suggests that height-growth patterns of lodgepole pine may differ by ecosystem. The most efficient method to test for these differences is through a special study designed specifically to test this hypothesis. The potential for differences in height-growth patterns is greatest on high sites at older ages. Thus a special study designed to test differences should be concentrated in older stands on high sites. If differences exist where the possibility is greatest, then a larger study could be designed to quantify differences across a wider range of ecosystems and site index. If differences do not exist or are not significant, then further work is not warranted and the cost of testing the hypothesis will be minimized.

High sites do not exist in all subzones thus possible differences could be tested for sites in the 20-m site-index class which occurs in a wide range of subzones. Differences could be tested in two subzones from different zones having contrasting climate, soil moisture, and nutrient regimes. At least 10 plots should be established in each subzone. Differences could be tested through analysis of residuals from curves fit to the plots in each group.

Height-growth patterns are usually compared using residual plots and statistics. However, this approach is often not adequate to detect differences in curve shape. Differences among ecosystems seem to exist in the timing and rate in which height-growth rates change. Some trees (at the same level of site index) appear to maintain the same pattern of height growth in early years, but differences become apparent around 70-80 years of

age. Traditional techniques do not detect these differences. Further work is needed to develop methods to characterize height-age curves. A possible approach would be to use the second and third derivatives of functions fit to individual trees and plots to characterize various curve shapes.

### **Old-Growth Stands**

The formulated height-age curves provide more accurate estimates of height growth before index age than the former curves. However, these data did not contain plots showing the pattern of height growth for older trees. The two plots greater than 130 years of age suggest that these curves may under-estimate height growth at older ages. This will affect growth and yield estimates and result in over-estimation of site index. Further sampling is required to better describe these height-growth patterns. This could be done as a supplement to this project where only older stands are sampled or in combination with studying the variation in height-growth among ecosystems. A separate study could be conducted by sampling 10 plots at ages greater than 150 years of age distributed evenly across the range of site index. This would probably be adequate to quantify the patterns of height growth at older ages. These plots could be combined with the data in this project and new curves formulated using the same procedures.

## **CONCLUSIONS**

These formulated height-age curves should provide reasonable estimates of the average dominant height of lodgepole pine for most ages and sites in B.C. The curves may slightly under-predict height at older ages, however, the difference and the effect on growth and yield should be very small. The curves provide the same estimates of height at older ages but are more accurate at younger ages than the currently used curves. We observed the same bias at early ages as the currently used curves when using the same model, thus this is probably the result of the inability of the model to fully describe early height-growth patterns. The formulated curves over-predict height at age zero but are unbiased by 5 years breast-height age. Further development is needed to more accurately describe the pattern of height-growth at older ages. This could be done separately or in combination with another study to examine differences in the height-growth patterns across different ecosystems.

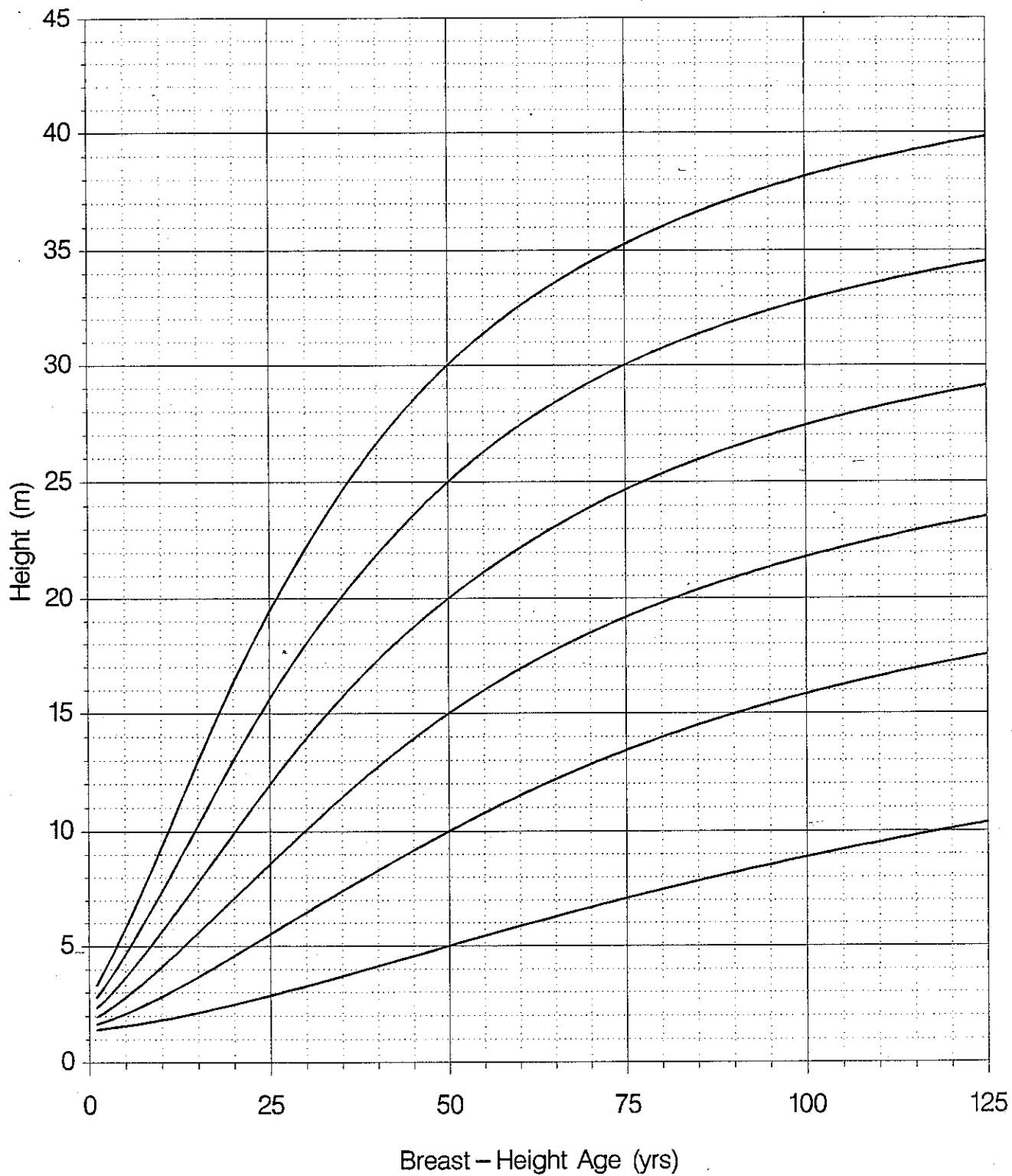
The formulated site-index curves give reasonably good estimates, however, no improvement was achieved over using the height-age curves to predict site index. The site-index curves may under-predict for trees greater than 100 years of age and for trees in the 30-m site index class. However, the bias should be small and less than the measurement error of height. The site-index curves have the advantage of providing a closed formulation where site index can be predicted directly. The height-age model provides equally good predictions that are compatible with height and age, however, site index must be predicted using iterative techniques.

## LITERATURE CITED

- Buford, M.A. and Burkhart, H.E. 1987. Genetic effects of growth and yield of loblolly pine plantations. *For. Sci.* 33:707-724.
- Carmean, W.H. 1972. Site index curves for upland oaks in the Central States. *For. Sci.* 18:102-120.
- Dahms, W.G. 1975. Gross yield of central Oregon lodgepole pine. *In* Management of lodgepole pine ecosystems. Proc. Symp. D.M. Braumgartner (ed.) Wash. State Univ., Coop Exten. Serv., Pullman WA. pp. 208-232.
- Ek, A. 1971. A formula for white spruce site index curves. Univ. Wisc. Madison, Wisc. For. Res. Note 161. 2 pp.
- Goudie, J.W. 1984. Height growth and site index curves for lodgepole pine and white spruce and interim managed stand yield tables for lodgepole pine in British Columbia. Contract Final Rep. Submitted to Min. For. Res. Br., Victoria, B.C. 75 pp.
- Goudie, J.W., Mitchell, K.J., and Polsson, K.R. 1990. Managed stand yield and product tables for interior lodgepole pine: Initial density and precommercial thinning. *Prov. B.C. Min. For. Res. Br., Victoria, B.C.* 106 pp. + app.
- Ministry of Forests. 1981. Site index equations and curves for the major tree species in British Columbia. *B.C. Min. For., Inv. Rep. No. 1.* 10 pp. + app.
- Ministry of Forests. 1991. Annual report 1989-1990. *Prov. of B.C. Min. For. Victoria, B.C.* 120 pp.
- Mitchell, K.J. 1975. Dynamics and simulated yield of Douglas-fir. *For. Sci. Monogr.* 17. 39 pp.
- Monserud, R.A. 1984. Height-growth and site index curves for inland Douglas-fir based on stem-analysis data and forest habitat type. *For. Sci.* 30:943-965.
- Nance, W.L. and Wells, O. 1981. Site index models for height growth of planted loblolly pine (*Pinus taeda* L.) seed sources. Pp. 86-96 *In* Proc. 16th S. For. Tree Impr. Conf., Blacksburg, VA.
- Payendeh, B. 1974. Nonlinear site index equations for several major Canadian timber species. *For. Chron.* 50:194-196.
- Thrower, J.S. and Goudie, J.W. 1992. Estimating dominant height and site index of even-aged interior Douglas-fir in British Columbia. *W. J. Appl. For.* 7:20-25.

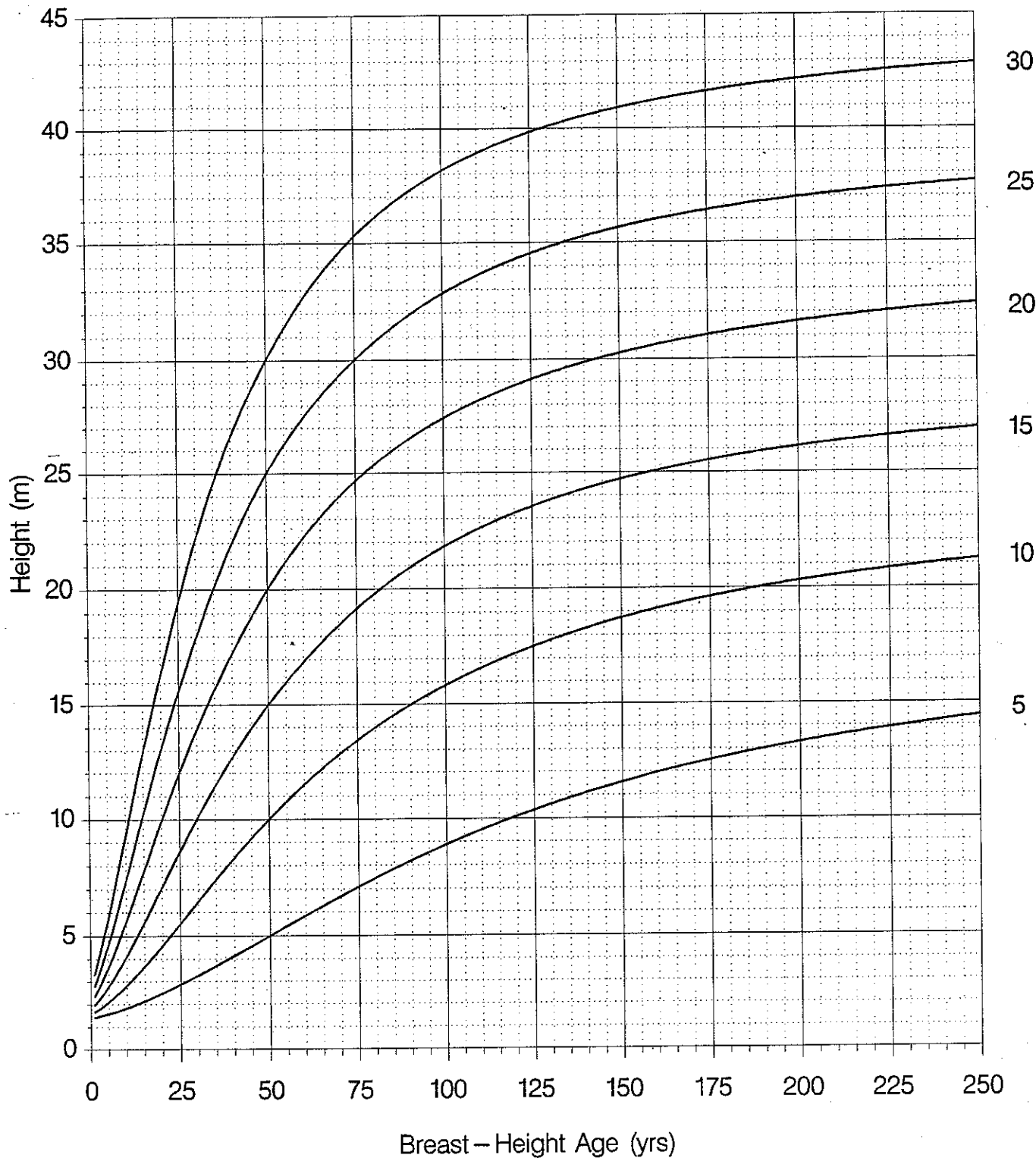
### Appendix I - Lodgepole Pine Height-Age Curves

SI



### Lodgepole Pine Height-Age Curves

SI



## Appendix II - Height by Site Index and Breast-Height Age

Table 4a. Predicted height (m) by site index (m) and breast-height age (yrs) from Eqn. 7 (Table 1 of 2).

Breast-Height Age	Site Index (m)										
	9	10	11	12	13	14	15	16	17	18	19
0	1.52	1.55	1.59	1.64	1.68	1.73	1.78	1.83	1.89	1.94	2.00
5	1.98	2.10	2.23	2.36	2.50	2.65	2.80	2.96	3.12	3.29	3.47
10	2.61	2.83	3.06	3.31	3.57	3.83	4.11	4.40	4.69	5.00	5.31
15	3.33	3.67	4.02	4.39	4.77	5.17	5.57	5.99	6.42	6.86	7.32
20	4.13	4.58	5.05	5.54	6.04	6.56	7.09	7.64	8.19	8.76	9.34
25	4.96	5.52	6.11	6.71	7.33	7.96	8.60	9.26	9.93	10.60	11.29
30	5.80	6.47	7.16	7.87	8.59	9.32	10.06	10.81	11.58	12.35	13.13
35	6.63	7.41	8.19	8.99	9.79	10.61	11.44	12.27	13.11	13.96	14.82
40	7.45	8.31	9.18	10.05	10.94	11.83	12.72	13.62	14.53	15.44	16.36
45	8.24	9.18	10.12	11.06	12.01	12.96	13.91	14.87	15.82	16.79	17.75
50	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00
55	9.72	10.78	11.83	12.88	13.92	14.96	16.00	17.03	18.06	19.10	20.12
60	10.40	11.51	12.60	13.69	14.77	15.84	16.91	17.97	19.03	20.08	21.13
65	11.05	12.19	13.32	14.44	15.55	16.64	17.74	18.82	19.90	20.97	22.04
70	11.66	12.83	13.99	15.13	16.26	17.38	18.49	19.59	20.68	21.77	22.85
75	12.22	13.43	14.61	15.77	16.92	18.05	19.18	20.29	21.40	22.49	23.58
80	12.76	13.98	15.18	16.36	17.52	18.67	19.80	20.93	22.04	23.15	24.24
85	13.26	14.50	15.71	16.90	18.08	19.23	20.38	21.51	22.63	23.74	24.84
90	13.72	14.98	16.20	17.41	18.59	19.75	20.90	22.03	23.16	24.27	25.38
95	14.16	15.42	16.66	17.87	19.05	20.22	21.38	22.51	23.64	24.76	25.87
100	14.56	15.84	17.08	18.30	19.49	20.66	21.81	22.95	24.08	25.20	26.31
105	14.95	16.23	17.47	18.69	19.88	21.06	22.22	23.36	24.49	25.61	26.72
110	15.30	16.59	17.84	19.06	20.25	21.43	22.58	23.73	24.86	25.98	27.09
115	15.64	16.93	18.18	19.40	20.59	21.77	22.93	24.07	25.20	26.32	27.43
120	15.95	17.24	18.49	19.71	20.91	22.08	23.24	24.38	25.51	26.63	27.74
125	16.24	17.53	18.78	20.00	21.20	22.37	23.53	24.67	25.80	26.92	28.03
130	16.52	17.81	19.06	20.28	21.47	22.64	23.80	24.94	26.07	27.18	28.29
135	16.77	18.06	19.31	20.53	21.72	22.89	24.05	25.19	26.31	27.43	28.54
140	17.02	18.30	19.55	20.77	21.96	23.13	24.28	25.42	26.54	27.66	28.76
145	17.24	18.53	19.78	20.99	22.18	23.35	24.50	25.63	26.75	27.87	28.97
150	17.46	18.74	19.99	21.20	22.38	23.55	24.70	25.83	26.95	28.06	29.17
155	17.66	18.94	20.18	21.39	22.57	23.74	24.88	26.02	27.14	28.25	29.35
160	17.85	19.13	20.37	21.57	22.75	23.91	25.06	26.19	27.31	28.42	29.51
165	18.03	19.30	20.54	21.74	22.92	24.08	25.22	26.35	27.47	28.57	29.67
170	18.20	19.47	20.70	21.90	23.08	24.24	25.38	26.50	27.62	28.72	29.82
175	18.36	19.63	20.86	22.05	23.23	24.38	25.52	26.65	27.76	28.86	29.96
180	18.51	19.78	21.00	22.20	23.37	24.52	25.66	26.78	27.89	28.99	30.09
185	18.65	19.92	21.14	22.33	23.50	24.65	25.78	26.90	28.01	29.12	30.21
190	18.79	20.05	21.27	22.46	23.62	24.77	25.90	27.02	28.13	29.23	30.32
195	18.92	20.17	21.39	22.58	23.74	24.89	26.02	27.13	28.24	29.34	30.43
200	19.04	20.29	21.51	22.69	23.85	25.00	26.12	27.24	28.35	29.44	30.53

Table 4b. Predicted height (m) by site index (m) and breast-height age (yrs) using Eqn. 7 (Table 2 of 2).

Breast- Height Age	Site Index (m)										
	20	21	22	23	24	25	26	27	28	29	30
0	2.06	2.13	2.19	2.26	2.33	2.40	2.47	2.55	2.63	2.71	2.79
5	3.65	3.84	4.03	4.23	4.43	4.64	4.86	5.08	5.30	5.54	5.77
10	5.63	5.97	6.31	6.66	7.01	7.38	7.75	8.13	8.52	8.91	9.32
15	7.78	8.25	8.74	9.23	9.73	10.24	10.76	11.29	11.83	12.38	12.93
20	9.93	10.53	11.14	11.76	12.39	13.03	13.67	14.33	14.99	15.66	16.34
25	11.99	12.70	13.42	14.14	14.88	15.62	16.37	17.12	17.89	18.66	19.43
30	13.92	14.71	15.52	16.33	17.15	17.97	18.80	19.63	20.47	21.32	22.17
35	15.68	16.55	17.42	18.30	19.18	20.07	20.96	21.85	22.75	23.65	24.56
40	17.28	18.20	19.13	20.06	20.99	21.92	22.86	23.80	24.75	25.69	26.64
45	18.71	19.68	20.65	21.62	22.59	23.56	24.53	25.51	26.48	27.46	28.44
50	20.00	21.00	22.00	23.00	24.00	25.00	26.00	27.00	28.00	29.00	30.00
55	21.15	22.18	23.20	24.22	25.25	26.27	27.29	28.31	29.32	30.34	31.36
60	22.18	23.23	24.27	25.31	26.35	27.38	28.42	29.45	30.48	31.51	32.54
65	23.10	24.16	25.22	26.27	27.32	28.37	29.41	30.46	31.50	32.54	33.57
70	23.93	25.00	26.06	27.13	28.19	29.24	30.30	31.35	32.39	33.44	34.48
75	24.67	25.75	26.82	27.89	28.96	30.02	31.08	32.13	33.19	34.24	35.28
80	25.33	26.42	27.50	28.57	29.64	30.71	31.77	32.83	33.89	34.94	35.99
85	25.93	27.02	28.11	29.18	30.26	31.33	32.39	33.45	34.51	35.57	36.62
90	26.48	27.57	28.65	29.73	30.81	31.88	32.95	34.01	35.07	36.13	37.19
95	26.97	28.06	29.15	30.23	31.31	32.38	33.45	34.51	35.58	36.63	37.69
100	27.41	28.51	29.60	30.68	31.76	32.83	33.90	34.97	36.03	37.09	38.14
105	27.82	28.92	30.00	31.09	32.17	33.24	34.31	35.37	36.44	37.49	38.55
110	28.19	29.29	30.38	31.46	32.54	33.61	34.68	35.74	36.81	37.86	38.92
115	28.53	29.63	30.71	31.80	32.87	33.95	35.02	36.08	37.14	38.20	39.26
120	28.84	29.94	31.02	32.11	33.18	34.25	35.32	36.39	37.45	38.51	39.56
125	29.13	30.22	31.31	32.39	33.47	34.54	35.60	36.67	37.73	38.79	39.84
130	29.39	30.48	31.57	32.65	33.72	34.80	35.86	36.92	37.98	39.04	40.09
135	29.63	30.73	31.81	32.89	33.96	35.03	36.10	37.16	38.22	39.28	40.33
140	29.86	30.95	32.03	33.11	34.18	35.25	36.32	37.38	38.44	39.49	40.54
145	30.07	31.16	32.24	33.32	34.39	35.46	36.52	37.58	38.64	39.69	40.74
150	30.26	31.35	32.43	33.50	34.58	35.64	36.71	37.77	38.82	39.88	40.93
155	30.44	31.53	32.61	33.68	34.75	35.82	36.88	37.94	38.99	40.05	41.10
160	30.61	31.69	32.77	33.84	34.91	35.98	37.04	38.10	39.15	40.21	41.26
165	30.76	31.85	32.92	34.00	35.07	36.13	37.19	38.25	39.30	40.35	41.40
170	30.91	31.99	33.07	34.14	35.21	36.27	37.33	38.39	39.44	40.49	41.54
175	31.04	32.13	33.20	34.27	35.34	36.40	37.46	38.52	39.57	40.62	41.67
180	31.17	32.25	33.33	34.40	35.46	36.52	37.58	38.64	39.69	40.74	41.79
185	31.29	32.37	33.45	34.51	35.58	36.64	37.70	38.75	39.80	40.85	41.90
190	31.41	32.48	33.56	34.62	35.69	36.75	37.80	38.86	39.91	40.96	42.00
195	31.51	32.59	33.66	34.73	35.79	36.85	37.90	38.96	40.01	41.05	42.10
200	31.61	32.69	33.76	34.82	35.89	36.94	38.00	39.05	40.10	41.15	42.19